

Least-squares shot-profile wave-equation migration

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Summary

We introduce *least-square shot-profile wave-equation migration*. Least-squares migration uses a forward operator (de-migration), and an adjoint operator (migration). We derive these operators using pre-stack split-step wave-field propagators, omitting some details for lack of space. We apply shot-profile least-squares migration to a synthetic example using data from the Sigsbee 2a model.

Introduction

Previously, Rickett (2003) described algorithms for shot-profile split-step de-migration and migration using operator notation. We provide a more explicit algebraic derivation of the operators. In the context of least-squares migration, these are, respectively, the adjoint and forward operators for the least-squares normal equations. The algorithm that solves the set of least-squares normal equations, we call least-squares shot-profile wave-equation migration.

Least-squares migration has received ample attention in the geophysical literature. Keys and Weglein (1983) introduced a generalized linear inversion for the Born approximation, allowing for incomplete data and prior information. Their paper presents the first non-direct (i.e. least-squares) solver for the Born approximation. Concurrently, authors discussed the Born approximation and its direct inverse solution using constant velocity Green's functions (e.g. Cohen and Bleistein, 1979) and WKB Green's functions (e.g. Clayton and Stolt, 1981). On a parallel tract, a succession of wave-field propagation techniques for migration developed, for example, allowing for migration velocity models that vary in depth only (Gazdag, 1978). A multitude of approximations allowing for laterally varying velocity models followed (e.g. Stoffa et al., 1990). With the privilege of hindsight, one may choose to take the perspective that these wave-field propagators are approximations to the Green's function in the Born approximation (e.g. Huang et al., 1999). This is the perspective that we take in this study.

In addition to the choice of propagator, one must choose a geometry for the seismic experiment, or equivalently the domain in which the migration algorithm operates. Within the regime of pre-stack migration there are two geometries commonly used, shot-receiver and shot-profile. Although the equivalence between the two geometries has been shown (Biondi, 2003), they differ in their respective parameterizations of the pre-stack migrated image gathers. From a practitioner's point of view, this difference in parameterization is important (Jeannot, 1988).

The first widely cited practical implementation of the ideas introduced by Keys and Weglein (1983) is Nemeth et al. (1999) which uses Kirchoff operators to propagate the wave-field. Later, Kuhl and Sacchi (2003) used wave equation based propagators. In both cases, the algorithms can be classified of type shot-receiver. These implementations validate the ideas in Keys and Weglein (1983) that, for example, generalized inversion can compensate for incomplete data.

Forward and adjoint operators

In the context of least-squares migration, the forward operator is wave-field modeling (often referred to as de-migration). Our construction of the forward operator uses 1) the Born

approximation, 2) a constant velocity Green's function, altered for laterally varying velocity using the split-step approximation, and 3) a Gazdag depth marching algorithm for depth varying velocity.

For the Born approximation to the wave-field, we write $\psi \approx \psi_d + \psi_s$, where ψ_d is the direct wave-field,

$$\psi_d(\mathbf{x}_g, z_g | \mathbf{x}_s, z_s; \omega) = f(\omega) G_0(\mathbf{x}_g, z_g | \mathbf{x}_s, z_s; \omega), \quad (1)$$

and the Born approximation for the scattered wave-field is ψ_s ,

$$\psi_s(\mathbf{x}_g, z_g | \mathbf{x}_s, z_s; \omega) = f(\omega) \iint G_0(\mathbf{x}_g, z_g | \mathbf{x}', z'; \omega) \left(\frac{\omega}{c_0(\mathbf{x}', z')} \right)^2 \alpha(\mathbf{x}', z') G_0(\mathbf{x}', z' | \mathbf{x}_s, z_s; \omega) d\mathbf{x}' dz', \quad (2)$$

where,

$$\alpha(\mathbf{x}', z') = 1 - c_0^2(\mathbf{x}', z') / c^2(\mathbf{x}', z'), \quad (3)$$

is called the scattering potential (e.g. Keys and Weglein et al, 1983). In equations 1-3, \mathbf{x}_g and \mathbf{x}_s are, respectively, lateral geophone and source positions, with z_g and z_s being their respective depths below the surface. The function $f(\omega)$ is the Fourier representation of the seismic source, and G_0 is a Green's function. Within the context of least-squares migration, α (with a chosen parameterization) are migrated images, c is the Earth's velocity, and c_0 is the migration velocity. We have assumed an acoustic and constant density wave equation.

To evaluate equations 1 and 2 for when the migration velocity c_0 varies in both its lateral and depth dimensions, we employ the wave-field propagator described in Gazdag (1978) along with the split-step approximation of Stoffa et al. (1990). Figure 1 illustrates the propagator for the demigration operator. The migration velocity is partitioned into layers, and within each layer the migration velocity is constant with respect to depth, but is allowed to vary in its lateral dimensions. The contribution to the scattered wave-field from each layer is computed using equation 2 and a boundary condition computed for its top surface. For the shallowest layer, the boundary condition is given by the location and frequency distribution of the seismic source. For deeper layers the boundary condition is computed using the direct wave-field (equation 1). The total measured (scattered) wave-field is $\sum_l \psi_{s(l)}$, where $\psi_{s(l)}$ is the contribution from the l^{th} layer.

For lack of space, we omit details of the derivation, giving only the final result for the forward operator. In particular, the sum over $\psi_{s(l)}$ can be expressed using two iterative methods, first for downward continuing the source side wave-field into the earth,

$$v_{s(l)}(\mathbf{x}_g, \omega; \mathbf{x}_s) = u_{s(l)} \mathbf{F}^* u_{p(l)} g(\mathbf{k}_{gx}, \omega; \mathbf{x}_s) \quad (4)$$

$$v_{s(l)}(\mathbf{x}_g, \omega; \mathbf{x}_s) = u_{s(l)} \mathbf{F}^* u_{p(l)} \mathbf{F} v_{s(l-1)}, \quad l = 2 \dots n,$$

and, second, for constructing the measured wave-field,

$$\psi_s(\mathbf{x}_g, z_0 | \mathbf{x}_s, z_0; \omega) = \Delta z v_1 \quad (5)$$

$$v_l(\mathbf{x}_g, \omega; \mathbf{x}_s) = u_{s(l)} \mathbf{F}^* u_{p(l)} \mathbf{F} (v_{s(l)} (2\pi)^{-4l} (\omega/c_{1(l)})^2 \alpha(\mathbf{x}_g, z_l) + v_{l+1}),$$

for $l = n \dots 1$, and $v_{n+1} = 0$ (n are the number of layers in the migration velocity model). In equations 4 and 5, $g(\mathbf{k}_{gx}, \omega; \mathbf{x}_s)$ is the frequency distribution of the seismic source at lateral shot position \mathbf{x}_s . Further, $u_{p(l)}$ and $u_{s(l)}$ are, respectively, phase shift and split-step operators for the

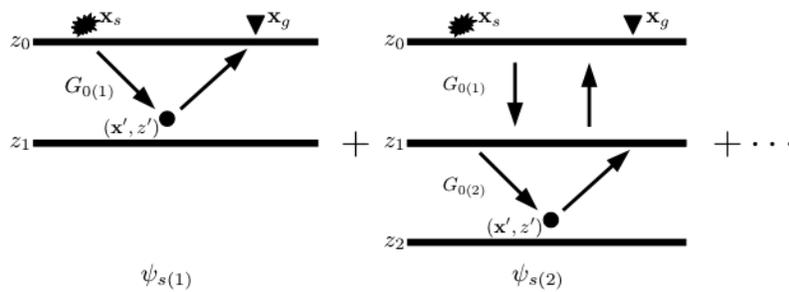


Figure 1: We give a schematic description of the first two terms in the series representation of the de-migration algorithm, where $G_{0(l)}$ is the split-step Green's function for the l^{th} layer.

l^{th} layer that arise from the split-step approximation to the Green's function. \mathbf{F} is the Fourier transform over \mathbf{x}_g , Δz is layer thickness, and $c_{1(l)}$ is an average migration velocity for the l^{th} layer.

The adjoint operator (migration) can be found by re-writing the forward operator (equations 4 and 5) in the form of a discretized Fredholm integral equation of the first kind which has a well known adjoint. This, in turn, results in the often-used split-step shot-profile migration algorithm, and that for lack of space, we omit from our discussion.

Shot-profile least-squares migration

We let \mathbf{d} be a vector realized from all recorded shot gathers $\psi_s(\mathbf{x}_g, z_0 | \mathbf{x}_s, z_0; \omega)$. Likewise, we let \mathbf{m} be a vector realized from the scattering potential $\alpha(\mathbf{x}_g, z; \mathbf{x}_s)$ such that for each shot location, its aperture can vary. Then, we let \mathbf{A} be the matrix built from the de-migration operator defined by equations 4 and 5, and \mathbf{A}^H its adjoint (migration operator). \mathbf{A} maps \mathbf{m} to \mathbf{d} , and to find optimal migrated images, we solve the set of least-squares normal equations,

$$(\mathbf{A}^H \mathbf{W}^H \mathbf{W} \mathbf{A} + \mu \mathbf{I}) \mathbf{m} = \mathbf{A}^H \mathbf{W}^H \mathbf{W} \mathbf{d}, \quad (6)$$

for \mathbf{m} . In equation 6, \mathbf{W} is a data weighting matrix allowing for incomplete data, and μ is a trade-off parameter. We solve equation 6 by the conjugate gradient method, and implicit construction of the matrices.

Example

For example, we consider the Sigsbee 2a model. We use finite-difference data generated by the Madagascar project. For this example, we use a single shot gather (not shown), illustrating the effectiveness of least-squares migration applied to a limited amount of data. The result of applying the migration and least-squares migration algorithms to the single shot gather are shown (for a short window in depth), respectively, in Figures 2a and 2b. In Figure 2c, we show the true scattering potential (akin to a reflectivity model).

The example shows subtle improvements in the migrated image when shot-profile least-squares migration is used in place of shot-profile migration. For example, the point diffractors located at approximately 5.2km in depth are better resolved.

Conclusions

We derive a least-square shot-profile migration algorithm using a split-step wave-field propagator. This differs from previously published *shot-receiver* least-squares migration algorithms, allowing for the (de-)migration operators to be applied one shot at a time, as well as giving an alternative parameterization of the migrated image gathers.

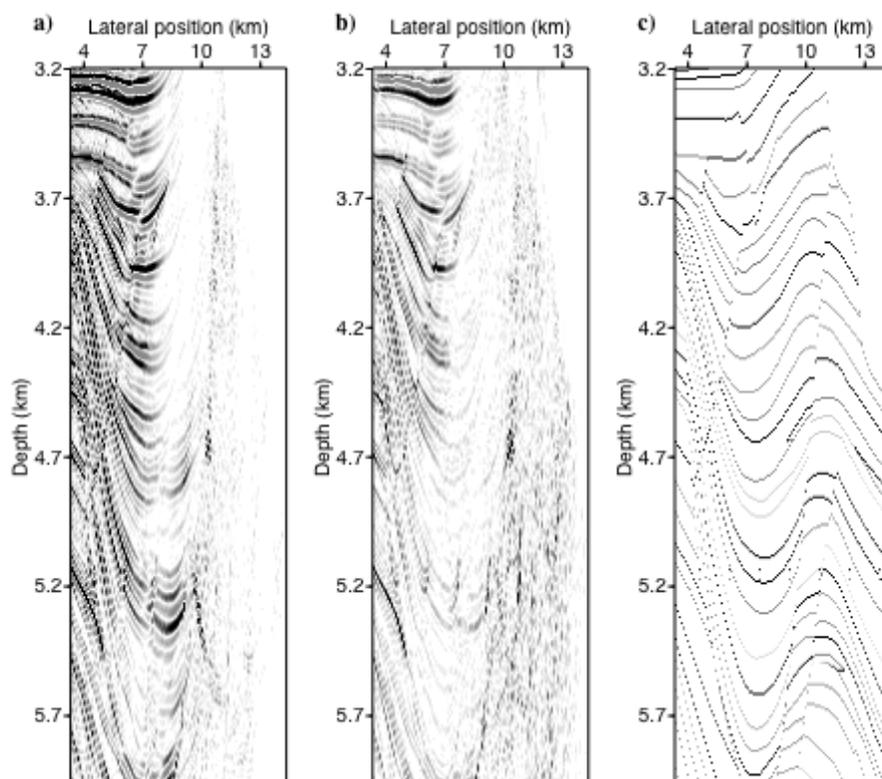


Figure 2: Sigsbee 2a example, single shot: a) the migration (adjoint), b) the least-squares migration (inverse), and c) the true reflectivity.

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References

- Biondi, B., 2003, Short note: equivalence of source-receiver and shot-profile migration: *Geophysics*, 68, 1340-1347.
- Clayton, R. W., and Stolt, R. H., 1981, A Born-WKB inversion method for acoustic reflection data: *Geophysics*, 46, 1559-1567.
- Cohen, J. K., and Bleistein, N., 1979, Velocity inversion procedure for acoustic waves: *Geophysics*, 44, 1077-1087.
- Gazdag, J., 1978, Wave equation migration with phase-shift method: *Geophysics*, 43, 1342-1351.
- Huang, L., Fehler, M. C., and Wu, R., 1999, Extended local Born Fourier migration method: *Geophysics*, 64, 1524-1534.
- Jeannot, J. P., 1988, Full prestack versus shot record migration: practical aspects: 58th Annual International Meeting, SEG, Expanded Abstracts, 966-968.
- Kuhl, H., and Sacchi, M. D., 2003, Least-squares wave-equation migration for AVP/AVA inversion: *Geophysics*, 68, 262-273.
- Keys, R. G., and Weglein, A. B., 1983, Generalized linear inversion and the first Born theory for acoustic media: *Journal of Mathematical Physics*, 24, 1444-1449.
- Nemeth, T., Wu, C., and Schuster, G. T., 1999, Least-squares migration of incomplete reflection data: *Geophysics*, 64, 262-273.
- Rickett, J. E., 2003, Illumination-based normalization for wave-equation depth migration: *Geophysics*, 68, 1371-1379.
- Stoffa, P. L., Fokkema, J. T., de Luna Freire, R. M. and Kessinger, W. P., 1990, Split-step Fourier migration: *Geophysics*, 55, 410-421.