Gabor Depth Migration using a New Adaptive Partitioning Algorithm

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In Gabor depth migration, the temporal Fourier transform is applied to the input seismic data to transform to the frequency domain. Then the spatial Fourier transform is applied to the wavefield localized in windows, which sum to one, known collectively as a *partition of unity*. This procedure is called the *windowed* Fourier transform, also termed as the Gabor transform. The Gabor-transformed wavefield is extrapolated in depth by spatially variable phase shift and the inverse Gabor transform gives the extrapolated wavefield at the new depth. Since each spatial window requires a separate Fourier transform, the number of windows should be minimized given an accuracy threshold. Adaptive partitioning algorithm can address this problem. We introduce a new adaptive partitioning algorithm using the lateral position error, estimated from the phase error, as a criterion. Examples of 1D partitioning and a Marmousi imaging result are demonstrated to illustrate the performance of Gabor depth migration with the new adaptive partitioning algorithm.

Introduction

Migration with *phase shift* (Gazdag, 1978) is an important migration tool. For a layered earth with laterally varying velocities, phase shift migration leads to other advanced methods, e.g., *phase shift plus interpolation* (PSPI) (Gazdag and Sguazzero, 1984). Stoffa et al. (1990) showed the split-step Fourier method, which augments phase shift by a laterally variable, wavenumber independent, phase shift. The "*phase screen*" migration method proposed by Wu et al (1992) extends the split-step Fourier concept to include wavenumber dependence in the laterally variable terms (e.g., Roberts et al., 1997; Rousseau and de Hoop, 2001; Jin et al., 2002). Margrave and Ferguson (1999) proposed *generalized phase shift plus interpolation* (GPSPI) and *nonstationary phase shift* (NSPS) theory and showed that GPSPI is the logical limit of PSPI, when a unique velocity is used for every spatial position. Outside exploration geophysics, GPSPI is referred to by Fishman and McCoy (1985) as the *locally homogeneous approximation* (LHA) of wave propagation at high frequencies. We adopt this term to put the GPSPI theory in a broader setting, emphasizing its physical meaning. Gabor depth migration (Grossman et al, 2002; Ma and Margrave, 2006) is closely related to, but different from the phase-screen method. This method uses the localizing windows (partitions) to break the input wavefield into pieces and then phase-shifts them to a new depth, where the pieces are reassembled giving the wavefield in the new depth. This process is carried on until a maximum desired depth is reached. We try to find optimal sets of windows at depths to improve the computation efficiency given lateral positioning errors of the wavefield. For simplicity, we describe imaging algorithm in 2D using the new adaptive partitioning.

Gabor Depth Migration Theory

The LHA wavefield extrapolator, also known as the GPSPI extrapolator, is formulated as (Margrave and Ferguson, 1999)

$$\psi(x, z+\Delta z, \omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{\psi}(k_x, z, \omega)\hat{W}(k(x), k_x, \Delta z)\exp(-ik_x x)\,\mathrm{d}k_x, \qquad (1)$$

where $\psi(x,z,\omega)$ is the seismic wavefield in the space-frequency domain at depth z, ω is temporal frequency, k_x and k_z (to be defined) compose the total wavenumber vector with a magnitude of $k(x)$ in the Fourier domain, x denotes transverse coordinates, $\hat{\psi}(k_x, z, \omega)$ is the Fourier transform of $\psi(x,z,\omega)$, Δz is the step size of extrapolation in z (vertical) direction, and $\hat{W}$ is the LHA wavefield extrapolator, which is a spatial phase shift term and defined as

$$\hat{W}(k(x), k_x, \Delta z) = \exp\left(ik_z(k(x), k_x)\Delta z\right) \qquad (2)$$

$$k_z = \begin{cases} \sqrt{k^2(x) - k_x^2}, & k^2(x) > k_x^2 \\ i\sqrt{k_x^2 - k^2(x)}, & k^2(x) < k_x^2 \end{cases}, \quad k(x) = \frac{\omega}{v(x)} \qquad (3)$$

where $v(x)$ denotes the lateral velocity at x along a slab with thickness Δz.

We approximate LHA extrapolator using

$$\hat{W}(k(x), k_x, \Delta z) \approx \sum_j \Omega_j S_j(x)\hat{W}(k_j, k_x, \Delta z), \qquad (4)$$

where $\{\Omega_j\}$ is a set of windows forming a partition of unity (POU), meaning $\sum_j \Omega_j = 1$, the split-step Fourier operator (Stoffa et al., 1990) is $S_j(x) = \exp\left(i\omega\Delta z\left(v^{-1}(x) - v^{-1}_j\right)\right)$ and $\hat{W}(k_j, k_x, \Delta z)$ is defined by equation (4) using $k(x) = k_j = \omega / v_j$ with the reference velocities v_j defined as the average velocity across window Ω_j. Equation (4) asserts that the LHA extrapolator can be approximated by a split-step Fourier extrapolator within each window. Using approximation (4) in (1), the LHA formula becomes

$$\psi(x, z+\Delta z, \omega) \approx \frac{1}{2\pi}\sum_j \Omega_j(x)S_j(x)\int_{-\infty}^{+\infty}\hat{\psi}(k_x, z, \omega)\hat{W}(k_j, k_x, \Delta z)\exp(-ik_x x)\,\mathrm{d}k_x, \qquad (5)$$

where Ω_j and $S_j(x)$ are k_x independent and taken out of the integrand. Equation (5) is the formula used in the Gabor depth migration in this paper.

Building Adaptive Partitions

The simplest implementation of (5) would be to have $\{\Omega_j(x)\}$ be a set of constant-width windows, called *atomic* windows, uniformly sitting along the x coordinate. However, this is generally inefficient as the atomic window width must be chosen to accommodate the most rapid velocity change and this may be completely unnecessary elsewhere. For efficiency, we can sum adjacent atomic windows to form molecules in regions where there is little velocity variation. For this purpose, a criterion is required to specify when atomic windows can be consolidated. There are several methods (Grossman et al., 2002; Ma and Margrave, 2005) available but their consolidation criteria are not very physical. In this paper, a new scheme is developed using “lateral position error” as a criterion. Considering a straight ray emanating at angle θ from a point (x, z) and traversing to $(x, \Delta z)$, we have $x = \Delta z \tan\theta$ and $k = k_x / \sin\theta = \omega / v$. Differentiating these results and rearranging gives $dx = \Delta z \sec^2 \theta dv \partial\theta / \partial v$ and $\partial\theta / \partial v = \sec\theta k_x / \omega$, from which we conclude

$$dv = \frac{\cos^3\theta}{\sin\theta} \frac{dx}{\Delta z} v. \tag{6}$$

Equation (6) relates the position error expected in a single depth step, dx, to a velocity error of dv. This formula can be used to build the desired adaptive partition, given choices for the allowable positioning error and the maximum scattering angle of interest. To specify reference velocities, v_1 is chosen as the most frequently occurring velocity in a given depth step (easily done from a histogram of the discretely sampled $v(x)$) and then dv_1 is obtained from (6) as $dv_1 = \alpha v_1$, where $\alpha = \cos^3\theta \sin^{-1}\theta dx / \Delta z$. Letting v_2 denote the next higher reference velocity and v_3 the next lower, we set the conditions

$$\left. \begin{aligned} v_1 - dv_1/2 = v_3 + dv_3/2 = v_3(1 + \alpha/2) \\ v_1 + dv_1/2 = v_2 - dv_2/2 = v_2(1 - \alpha/2) \end{aligned} \right\} \tag{7}$$

Using (7), calculation of v_2 and v_3 gives

$$\left. \begin{aligned} v_2 = (2v_1 + dv_1)/(2 - \alpha) \\ v_3 = (2v_1 - dv_1)/(2 + \alpha) \end{aligned} \right\} \tag{8}$$

Figure 1 illustrates this process.

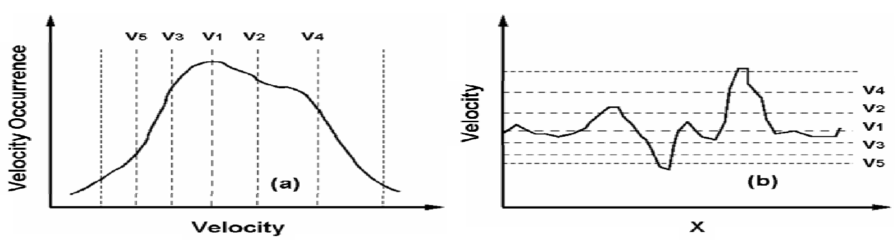

Figure 1. (a) The reference velocities chosen according to equation (6) are shown on top of a velocity histogram. (b) Velocity profile $v(x)$ (solid curve) and the reference velocities.

Proceeding until a reference velocity exceeds the range of $v(x)$, which defines a complete set of reference velocities. Once selected, they can be used as measures to build an indicator function set $\{I_j(x)\}$, using

$$I_j(x) = \begin{cases} 1, & |v(x) - v_j| = \min \\ 0, & \text{otherwise,} \end{cases} \tag{9}$$

where $v_j(j = 1, 2, ..., n)$ is an element in the reference velocity set $\{v_j\}$ created by (6). We convolve $I_j(x)$ with the atomic window $\Omega(x)$ to create the adaptive POU $W_j(x)$ by

$$W_j(x) = (I_j * \Omega)(x). \tag{10}$$

Thus there will be one window for each reference velocity even if the regions where that velocity is required are disjoint. Although we do not discuss it here, this method extends easily to 2D.

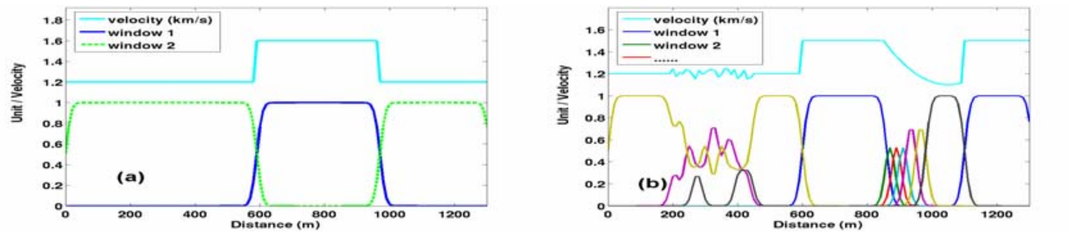

Figure 2. Non-uniform partitions of unity as created by the algorithm described here.

In Figure 2 (a), a bump velocity profile (shown in cyan colour) is used; two reference velocities are selected. Accordingly, two windows (green-dashed and blue-solid lines) have been chosen given a lateral position error criterion. Figure 2 (b) shows the partitioning algorithm can deal with such complicated lateral velocity variations as those in the Marmousi velocity model; a specific colour shows a single partition created by the algorithm.

Gabor Depth Migration Example

The Marmousi velocity section used in Gabor depth imaging in this paper is shown in Figure 3 (a). All depth images have been created using prestack shot migration.

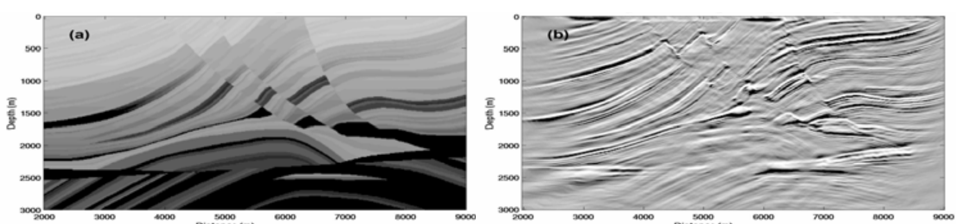

Figure 3. (a) Marmousi velocity. (b) Gabor depth migration result (a lateral position error of 5 m).

Figure 3(b) shows the prestack, shot-record, depth migration of the Marmousi dataset using a position-error criterion of 5 m. Taking a closer look at the imaging target (a reservoir extending from distance 6000 m to 7500 m in depth about 2500 m, see also Figure 3 (a)), we know that the target reservoir has been accurately imaged. Observing the rest of the areas in the image, we see that they are all well imaged.

Conclusions

The approximation of LHA extrapolator using the adaptive Gabor depth migration algorithm gives good results. The adaptive partitioning algorithm helps to achieve efficient depth imaging and the imaging accuracy and speed can be controlled using the adaptive partitioning scheme.

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