Computations of Circular Wavefront for Gridded Traveltime
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Introduction

Traveltime computations are an integral part of modelling and imaging seismic data by providing efficient kinematic information on the location of propagated energy. The traveltimes may be computed analytically using simplifying assumptions, or may be computed on a complex geological structure using raytracing or gridded traveltimes. A basic requirement for the propagation of gridded traveltimes is the estimation of one point on a corner of a square when given the traveltimes of the other three points on the square. A number of solutions are available to solve for the unknown time and are based on either a plane-wave assumption, a finite difference solution to the Eikonal equation, or an assumption that the wavefront at the square is curved. A known solution for estimating the center of curvature for a curved wavefront requires solving a quartic equation and choosing one of four possible solutions. An alternate method is presented to estimate the center of curvature using an iterative procedure that does not require solving the quartic equation.

Principle

The problem of estimating the center of curvature is illustrated in Figure 1a that shows one square of the grid with time $t_1$ at the origin, and the center of an equivalent circular wavefront at $(x_0, z_0, t_0)$. Each side of the square has a dimension $h$, and the velocity $v$ that is unique to this square, is assumed to occupy the entire space that includes the center of curvature. We desire to know the traveltime at $t_4$, which requires an estimation of $t_0$ and location of the source. Note that $t_0$ is not necessarily zero in areas with a complex wavefront due to varying velocities. Traveltime differences ($t_{21} = |t_2 - t_1|$ and $t_{31} = |t_3 - t_1|$) may be used to define two sets of hyperbolae that are symmetric about the $z = 0$ and $x = 0$ axis. These hyperbolae are shown in Figure 1b along with the corners of the square and four intersections, all of which are possible source locations. A well known solution to this problem (Vidale 1988) requires solving a quartic equation and choosing one solution from the four possible choices.

Solving the quartic equation can be difficult, but four straight forward analytic solutions can be obtained using symbolic math in Mathematica. These equations occupy 100 lines of text, (simplifications are possible) that require significant computation time, and involve complex arithmetic that also slows the speed of the computation.

Figure 1 Three point problem showing in a) the geometry of an assumed source and b) the two sets of hyperbolae that result from two time differences.
The new approach uses the three distances \( d_1, d_2, \) and \( d_3 \) from the geometry of Figure 1a, to define an equation that contains only one variable \( t_0 \). This equation

\[
t_0 = t_1 - \frac{1}{v} \sqrt{\left( \frac{v^2}{2h} \left( 2t_2 (t_2 - t_1) - t_2^2 + t_1^2 \right) + \frac{h}{2} \right)^2 + \left( \frac{v^2}{2h} \left( 2t_0 (t_3 - t_1) - t_3^2 + t_1^2 \right) + \frac{v^2}{2h} \right)^2},
\]

can be solved using an iterative technique based on the Newton-Raphson method. Once the value of \( t_0 \) is estimated, the estimated center of curvature \((x_0, z_0)\) is computed. Applications of this three point solution usually assume that the center of curvature is located in the third quadrant of Figure 1a, where this iterative technique provides a single solution.

Three methods for estimating \( t_4 \) are compared in Figure 2. A grid of spherical source points were defined in the third quadrant, and the three traveltimes \( t_1, t_2, \) and \( t_3 \) computed. The errors in estimating \( t_4 \) are compared using (a) the plane-wave assumption, (b) the Vidale finite difference method, and (c) the new iterative method. The source points are only located up to five square dimensions \( 5h \) away, and the maximum error is limited to ten percent. As expected in this region, the first two methods show significant error when compared with the iterative method.

![Plots](image1.png)

Figure 3  Comparison of errors in estimating the time \( t_4 \) using a) a plane wave assumption, b) Vidale’s method, and c) the new iterative method.

Reference