3-D TTI Eikonal Traveltime Kirchhoff Migration

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Introduction

Computing Kirchhoff migration traveltimes in anisotropic media is expensive since each propagation step involves solving an eigenvalue problem (Cerveny, 1972). Anisotropy is usually weak, however, allowing us to use a fast approximation derived by perturbing a simple, if unrealistic, elliptical anisotropic model (Ettrich and Gajewski, 1998; Alkhalifah, 2002). This approximation is especially critical in 3-D prestack imaging where data sizes can be very large.

To compute traveltimes in transversely isotropic media with a vertical symmetry axis (VTI anisotropy), Alkhalifah (2002) proposed a finite-difference algorithm based on perturbing the VTI anisotropic eikonal equation starting from an elliptically anisotropic background velocity model. This takes advantage of an expanding wavefront upwind finite-difference scheme with superior stability and accuracy properties.

Often, however, the VTI assumption is not well satisfied. If the sedimentary layering is not horizontal, for example, the symmetry axis of transverse isotropy is most likely tilted, and the angle-dependent velocity will be affected accordingly. The VTI assumption in these areas can cause significant imaging errors, resulting in mispositioning and misfocusing of exploration targets.

Where the VTI assumption is inadequate, we can instead assume a TTI media, which is transversely isotropic about a tilted symmetry axis. TTI is a common feature of shale formations in overthrust areas such as the Canadian Foothills.

In this paper I extend Alkhalifah's VTI traveltime algorithm to TTI media. I begin by deriving a linearized anisotropic eikonal equation for TTI media. Similarly an iterative linearization of the eikonal equation is used as the basis for an algorithm of finite-difference traveltime computations, starting with a background velocity model that is elliptically anisotropic in TTI media. Instead, however, of using vertical and NMO velocities, and the anisotropic coefficient η, to describe VTI anisotropic media, I use Thomsen's parameters υ_0, ε, and δ in TTI anisotropic media (Thomsen, 1986). Furthermore, rather than perturbing the horizontal velocity or the anisotropic parameter η, I perturb the anisotropy parameter δ since it is the least sensitive parameter in anisotropic media. Therefore only a single iteration is usually needed for a sufficiently accurate traveltime computation.

Theory

Begin with the eikonal equation for a VTI anisotropic media (Alkhalifah, 2002)

$$(1+2\varepsilon)\left(\left(\frac{\partial\tau}{\partial x}\right)^2+\left(\frac{\partial\tau}{\partial y}\right)^2\right)+\left(\frac{\partial\tau}{\partial z}\right)^2\left[1-2(\varepsilon-\delta)\upsilon_0^2\left(\left(\frac{\partial\tau}{\partial x}\right)^2+\left(\frac{\partial\tau}{\partial y}\right)^2\right)\right]=\frac{1}{\upsilon_0^2} \tag{1}$$

where $\tau(x, y, z)$ is the eikonal traveltime from the source to the point at coordinates (x, y, z), and υ_0, ε, and δ are Thomsen's parameters (Thomsen, 1986) at that point. This equation is fourth order in τ, as opposed to second order for the isotropic eikonal equation, complicating the finite-difference solution.

The idea is to linearize equation (1) with respect to δ. Begin by defining the residual traveltime $\Delta = \tau - \tau_n$ and the residual δ parameter $\lambda = \delta - \delta_n$. Substituting these into equation (1) and neglecting higher-order terms gives

(2)

$$\left(\frac{\partial \tau_n}{\partial x}\frac{\partial \Delta}{\partial x} + \frac{\partial \tau_n}{\partial y}\frac{\partial \Delta}{\partial y}\right)\left((1+2\varepsilon) - 2(\varepsilon - \delta_n)\upsilon_0^2\left(\frac{\partial \tau_n}{\partial z}\right)^2\right) +$$

$$\frac{\partial \tau_n}{\partial z}\frac{\partial \Delta}{\partial z}\left[1 - 2(\varepsilon - \delta_n)\upsilon_0^2\left(\left(\frac{\partial \tau_n}{\partial x}\right)^2 + \left(\frac{\partial \tau_n}{\partial y}\right)^2\right)\right] +$$

$$(\delta - \delta_n)\,\upsilon_0^2\left(\left(\frac{\partial \tau_n}{\partial x}\right)^2 + \left(\frac{\partial \tau_n}{\partial y}\right)^2\right) = 0$$

which is the iterative linearization of the eikonal equation (1). δ_n can be computed from equation (1) when an estimated traveltime τ_n is available. If we further assume that the initial traveltime τ_0 is obtained with equation (1) in an elliptically anisotropic media (that is, $\delta = \varepsilon$), then equation (1) becomes

(3)

$$(1+2\varepsilon)\left(\left(\frac{\partial \tau_0}{\partial x}\right)^2 + \left(\frac{\partial \tau_0}{\partial y}\right)^2\right) + \left(\frac{\partial \tau_0}{\partial z}\right)^2 = \frac{1}{\upsilon_0^2}$$

Substituting $\delta_0 = \varepsilon$ in equation (2), the first-order linearized eikonal equation simplifies to

(4)

$$(1+2\varepsilon)\left(\frac{\partial \tau_0}{\partial x}\frac{\partial \Delta}{\partial x} + \frac{\partial \tau_0}{\partial y}\frac{\partial \Delta}{\partial y}\right) + \frac{\partial \tau_0}{\partial z}\frac{\partial \Delta}{\partial z} +$$

$$(\delta - \varepsilon)\,\upsilon_0^2\left(\left(\frac{\partial \tau_0}{\partial x}\right)^2 + \left(\frac{\partial \tau_0}{\partial y}\right)^2\right) = 0$$

Now consider a transversely isotropic medium with a tilted symmetry axis dipping at an angle θ from the vertical direction along the azimuth angle ϕ from the inline direction. The elliptical anisotropic eikonal equation (3) and the first-order linearized eikonal equation (4) for this tilted media become

(5)

$$\alpha_1\left(\frac{\partial \tau_0}{\partial x}\right)^2 + \alpha_2\left(\frac{\partial \tau_0}{\partial y}\right)^2 + \alpha_3\left(\frac{\partial \tau_0}{\partial z}\right)^2 +$$

$$\beta_1\left(\frac{\partial \tau_0}{\partial x}\frac{\partial \tau_0}{\partial y}\right) + \beta_2\left(\frac{\partial \tau_0}{\partial y}\frac{\partial \tau_0}{\partial z}\right) + \beta_3\left(\frac{\partial \tau_0}{\partial z}\frac{\partial \tau_0}{\partial x}\right) = \frac{1}{\upsilon_0^2}$$

(6)

$$\gamma_1\left(\frac{\partial \Delta}{\partial x}\right) + \gamma_2\left(\frac{\partial \Delta}{\partial y}\right) + \gamma_3\left(\frac{\partial \Delta}{\partial z}\right) + \gamma_0\,(\delta - \varepsilon)\,\upsilon_0^2 = 0$$

where all coefficients $\alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \gamma_0, \gamma_1, \gamma_2,$ and γ_3, are functions of angles θ and ϕ and anisotropic parameters ε and δ. Coefficients $\gamma_0, \gamma_1, \gamma_2,$ and γ_3 also depend on the derivatives of the initial traveltime fields.

The TTI elliptical anisotropic eikonal equation (5) is similar to VTI anisotropic eikonal equation (3) except for the presence of additional traveltime cross derivatives resulting from the angular dependence of the anisotropy. Therefore, equation (5) can be solved numerically using the efficient expanding wavefront method developed for isotropic media (Qin et al, 1992; Sethian and Popovici, 1999). The cross derivatives, however, require a higher-order upwind finite-difference scheme.

The first-order linearized eikonal equation (6) is less complicated than the elliptical anisotropic eikonal equation (5) and can also be solved numerically using the efficient wavefront expanding method with a first-order upwind finite-difference scheme. This means that the linearized eikonal equation is also solved by computationally propagating along grids of minimum traveltimes. Since coefficients $\gamma_0, \gamma_1, \gamma_2$, and γ_3 are all functions of traveltime derivatives, stable space-domain convolution operators (Thurston and Brown, 1994) are used to calculate the derivatives of the traveltime field.

Examples

To demonstrate our algorithm for prestack depth migration I calculate the traveltimes for simple VTI and TTI models. The model consists of an anisotropic layer with constant parameters υ_0 = 5000 m/s, ε = 0.2, δ = 0.1, θ = 45 deg, and ϕ = 0 deg overlaying an isotropic half-space with a constant velocity of 6500 m/s. The input trace is located at (0,0,0). Figures 1 and 2 show the migration responses in the inline (y = 0) and crossline (x = 0) directions, respectively. Figure 3 shows the time slices at 600 ms. From the responses we see the influence of the tilted axis on migrations. During the presentation I will also show examples using physical models and real data.

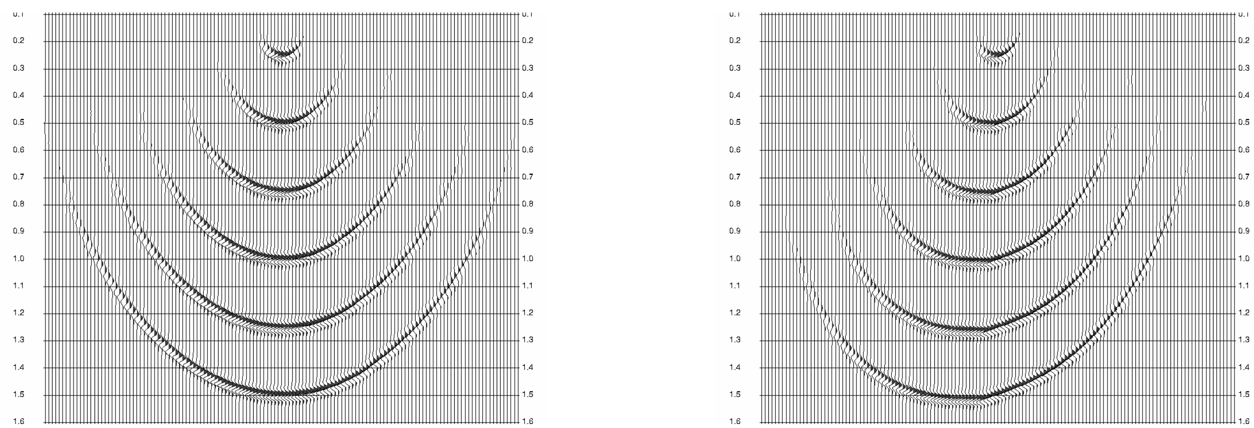

Figure 1: Migration impulse responses in the inline $(y = 0)$ from simple VTI (left) and TTI (right) models.

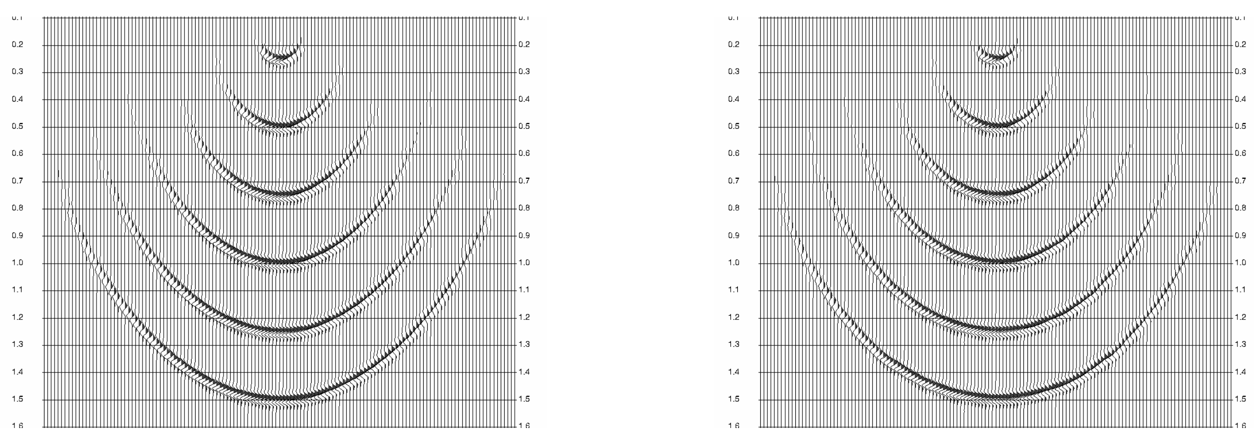

Figure 2: Migration impulse responses in the crossline $(x = 0)$ from simple VTI (left) and TTI (right) models.

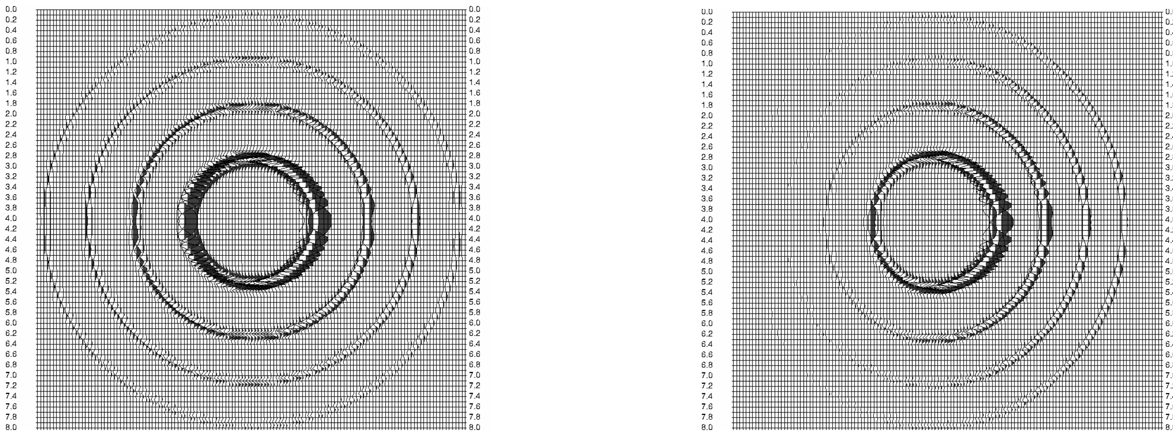

Figure 3: Time slices of migration impulse response at 600 ms from simple VTI (left) and TTI (right) models.

Conclusions

I have developed an algorithm for finite-difference traveltime computation, based on an iterative linearization of an anisotropic eikonal equation in transversely isotropic media with a tilted symmetry axis (TTI). I derive this by linearizing with respect to the anisotropy parameter δ , thereby reducing the number of iterations required. For most imaging applications, a single iteration is sufficient to give sufficient accuracy in traveltime computations.

Tests show that applying VTI assumption where tilt is actually non-zero will introduce significant errors in the migration because the tilt axis of symmetry has a significant effect on the traveltime calculations.

Future work will include computing branches of the multi-valued traveltime other than the first arrival (for example, the maximum energy) and coupling traveltime and amplitude computations.

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