Converted wave true amplitude prestack Kirchhoff migration

Xiaogui Miao, Sam Gray, Yu Zhang and Robert Kendall, Veritas DGC Inc.

Summary

Preserving amplitudes in converted wave prestack migration is important for subsequent AVO processing and interpretation. However, it has not received as much attention as amplitude preservation for P wave migration. In this paper, we derive common offset true amplitude weight functions in a v(z) medium for two and one half, and three dimensional converted wave Kirchhoff migration. The weight formulas are then simplified into the terms invovling travel time, ray path and velocity functions for efficient computation. The simplified weight functions are also examined with numeric modeling data for depth migration, which demonstrated well-controlled amplitude behavior in offset and depth.

Introduction

As multi component land and OBC data acquisition become more widely available, converted wave prestack migration is becoming a more routine process in seismic exploration and development. Since there are unavoidable topographic variations in land data and rough sea floors in OBC experiments, converted wave prestack Kirchhoff migration is still an important technique in prestack imaging. Besides, converted wave data contain additional information about velocities (Vp and Vs) and other rock properties. Especially, they tend to be more sensitive to the variations of anisotropic parameters than P waves (Jin, 2003). Extracting this information from converted wave sections by means of PS AVO, joint PP/PS inversion, and other interpretive processing has attracted some attention; in addition it has placed a critical requirement for preservation of amplitude information in processes such as prestack migration. For the single mode case, Bleistein (1987) developed a full range of theoretical formulas for two, two and one half, and three dimensional Kirchhoff migration. Several authors (Schleicher et al, 1993, Hanitzsch 1997, and Bleistein et al 2001) also presented "true amplitude" Kirchhoff migration weight functions. Since their weight functions involve square roots, numerical derivatives, and/or ray quantities, direct implementations are computational expensive. Dellinger et al (2000) first suggested a simplified two and one half dimensional weight function for constant velocity. Zhang et al (2000) then presented exact weight functions for two, two and one half, and three dimensional common shot, common receiver and common offset Kirchhoff migration with greatly simplified implementation formulas for a v(z) medium. However, for the multi-mode case, few papers address the true amplitude issue for prestack Kirchhoff migration because of complications due to asymmetric ray paths.

In this paper, we take 3D as an example and present a true amplitude weight function in a v(z) medium for common offset Kirchhoff migration, then follow Zhang's method (2000) to simplify the weight functions to reduce the computational load. The 2.5 D weighting function can be obtained following the same procedure. Our migration weight functions are then investigated with numerical model data for both 2.5 D and 3D cases.

Theory and method

According to Bleistein et al (2001), in the frequency domain a 3D prestack Kirchhoff migration can be presented as

$$\beta(\vec{x}) \sim \frac{1}{8\pi^2}\int d^2\vec{\xi}\,\frac{|h(\vec{x},\vec{\xi})|}{a(\vec{x},\vec{\xi})\left|\nabla\phi(\vec{x},\vec{\xi})\right|^2}\int i\omega d\omega e^{-i\omega\phi(\vec{x},\vec{\xi})}u_s(\vec{x}_r,\vec{x}_s,\omega)$$

where $\frac{|h(\vec{x},\vec{\xi})|}{a(\vec{x},\vec{\xi})\left|\nabla\phi(\vec{x},\vec{\xi})\right|^2}$ forms a migration weight function to produce a true amplitude image. In the migration weight, $\left|\nabla\phi(\vec{x},\vec{\xi})\right| = \left|\nabla(\tau_s+\tau_r)\right|$ is the travel time gradient, $a(\vec{x},\vec{\xi}) = A(\vec{x},\vec{x}_s(\vec{\xi}))A(\vec{x}_r(\vec{\xi}),\vec{x}\ )$, where $A(\vec{x},\vec{x}_s(\vec{\xi}))$ is the amplitude of Green's function with source at $\vec{x}_s$ and observation point at $\vec{x}$, and $A(\vec{x}_r(\vec{\xi}),\vec{x})$ is the counterpart with source at $\vec{x}$ and receiver at $\vec{x}_r$, $\left|h(\vec{x},\vec{\xi})\right|$ is the Beylkin determinant, which is the Jacobian to transform subsurface coordinates to surface coordinates. The Beylkin determinant is given by

$$h(\vec{x},\vec{\xi}) = \det\begin{vmatrix} \nabla\phi(\vec{x},\vec{\xi}) \\ \dfrac{\partial}{\partial\xi_1}\nabla\phi(\vec{x},\vec{\xi}) \\ \dfrac{\partial}{\partial\xi_2}\nabla\phi(\vec{x},\vec{\xi}) \end{vmatrix}.$$

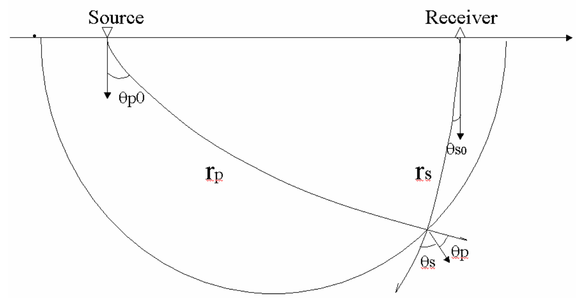


Figure 1, the scattering angles for PS wave.

For different configurations, common shot, common receiver and common offset, the weight functions are different. In this paper we take the common offset case as an example, since it is a combination of common shot and common receiver cases.

In the single mode v(z) medium for common offset configuration , we have (Zhang et al , 2000)

$$a(\vec{x},\vec{\xi}) = \sqrt{\frac{v_s v_{s0}}{\cos\theta_s\sigma_s\psi_s}}\sqrt{\frac{v_r v_{r0}}{\cos\theta_r\sigma_r\psi_r}} \approx \frac{1}{r_s r_r}.$$

This is also true for converted waves with $a(\vec{x},\vec{\xi}) \approx \frac{1}{r_p r_s}$. However, the relation $\left|\nabla\phi(\vec{x},\vec{\xi})\right| = \frac{2}{v}\cos\theta$, where $2\theta = \theta_r + \theta_s$ is the angle between a specular ray pair for the single mode, will be no longer hold for converted waves. Since for converted waves, the specular source and receiver pair travels with P-wave velocity down and S-wave velocity up (see Figure 1), the above relationship must be changed to

$$|\nabla\phi|^2 = (\nabla\tau_p + \nabla\tau_s, \nabla\tau_p + \nabla\tau_s) = (\frac{1}{v_p^2}(\gamma^2 + 1 + 2\gamma\cos(\theta_p + \theta_s)),$$

where γ =Vp/Vs. The Beylkin determinant also becomes more complicated than the single mode case. In a v(z) medium for converted waves it is given as:

$$h = \frac{1}{v_p}\left(\frac{1+\gamma\cos(\theta_p+\theta_s)}{\psi_p\cos\theta_p} + \frac{\gamma+\cos(\theta_p+\theta_s)}{\psi_s\cos\theta_s}\right)\left(\frac{1}{\sigma_p}+\frac{1}{\sigma_s}\right)$$
$$+\frac{H^2}{v_p\sigma_p\sigma_s\psi_p\psi_s}\left[\frac{(\psi_s-\sigma_s)(\cos^2\theta_p\psi_p-\sigma_p)}{\cos\theta_p} + \gamma\frac{(\psi_p-\sigma_p)(\cos^2\theta_s\psi_s-\sigma_s)}{\cos\theta_s}\right]$$

where

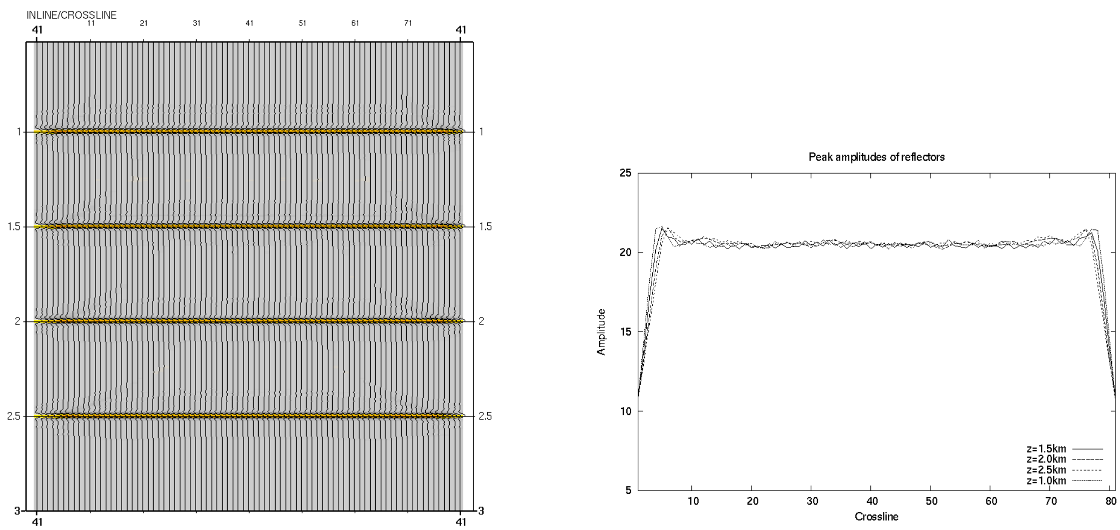
$$H = \frac{(x-x_p)(y-y_s)-(y-y_p)(x-x_s)}{\sqrt{(x-x_p)^2+(y-y_p)^2}\sqrt{(x-x_s)^2+(y-y_s)^2}}$$

$$\sigma_p = \int_0^z \frac{v_p(\zeta)}{\cos\theta_p(\zeta)}d\zeta,\ \psi_p = \int_0^z \frac{v_p(\zeta)}{\cos^3\theta_p(\zeta)}d\zeta.$$

Here σ_p and ψ_p are P wave in-plane and out-of-plane geometrical spreading terms. σ_s and ψ_s are defined similarly for the S-wave parameters. To compute the migration weight using this formula directly is very expensive. Zhang et al. (2000) introduced a simplification method to approximate the weight function for the single mode case in order to reduce the computational load, and demonstrated that the simplified weights produced comparable results to analytic solutions. Here, we follow the same practise and simplify the above weight function for converted wave migration. The simplified version is much more efficient to compute since it only involves travel time, velocities and ray paths, all of which can be stored in the travel time tables and calculated on a coarse grid and interpolated to a finer migration grid. Following the same procedure we have also derived a migration weight function for the 2.5D case of a v(z) medium. The above method can be easily extented to an anisotropic medium, since the required information to calculate weight functions is all carried in the travel time tables and velocities. The program is implemented on PC clusters, and we have observed no significant additional computation time compared with the conventional kinematic converted wave prestack Kirchhoff migration.

Numerical modeling examples

In order to verify our amplitude weights, we created a 3D numerical model data set, which has four flat events at depths of 1km, 1.5km, 2km and 2.5km. To be able to use analytic formulas to generate synthetic converted-wave data with correct amplitudes including geometric spreading, we assume that the velocities are constant at Vp=3000m/s and Vs=2000m/s, the reflectivities are due to density changes, and the four events at different depths have the same reflectivities. Migration attempts to recover all the amplitudes. Therefore, after migration we should produce an image with four reflectors having the same ampltiudes at all the depths.



(a) (b)

Figure 2. (a) Converted wave migrated section of a 3D synthetic model with four reflections at depths of 1.0, 1.5, 2.0 and 2.5km, (b) the peak amplitudes picked from the four reflections in the left section.

Figure 2a shows the converted wave migrated section. From the migrated image we can see that all the reflections have demonstrated almost uniform amplitudes. To check the details of the amplitudes we have picked peak amplitudes for all four events at different depths and ploted them in Figure 2b. Apart from edge effects and some jitter caused by interference with artifacts and slightly uneven fold in the geometry for each depth at each converted wave image point (remember, unlike the single mode case, the converted wave imaging point varies along the depth for each location), the migrated image shows well controlled amplitude behavior and all the four reflections have amplitudes in the range of 20 to 21.

Since 2.5D ray theory and migration characterize in plane propagation of wave form from a point source in 3D, they more closely represent reality than the 2D in plane propagation that does not recognize the out of plane geometrical spreading of waves. Therefore, we have implemented the 2.5D method for our 2D migration. To see which weight function preserves amplitude better (2D or 2.5D), a synthetic model of two reflections of the same reflectivities was created with one flat event at 1500 meters and another 15 degree dipping event. Results are shown in Figure 3. Comparing the common image gathers at image point 230 for 2D and 2.5D (see Figure 3b and 3c), we can see that across the offset range 0 to 3000m for the flat event at 1500m depth both 2D and 2.5D weights display well controlled amplitudes. However, for the dipping event between 2100m and 2200m at the same image location, the 2.5D amplitude weight properly recovers the amplitude loss during propagation, and displays consistent amplitudes compared with the flat event (see Figure 3b). The same amplitude behavior can be observed in the near offset migrated section in Figure 3a for 2.5D weight. There is obvious amplitude decay compared with the flat event for 2D weight due to failure to recover out of plane energy for the dipping event (see Figure 3c).

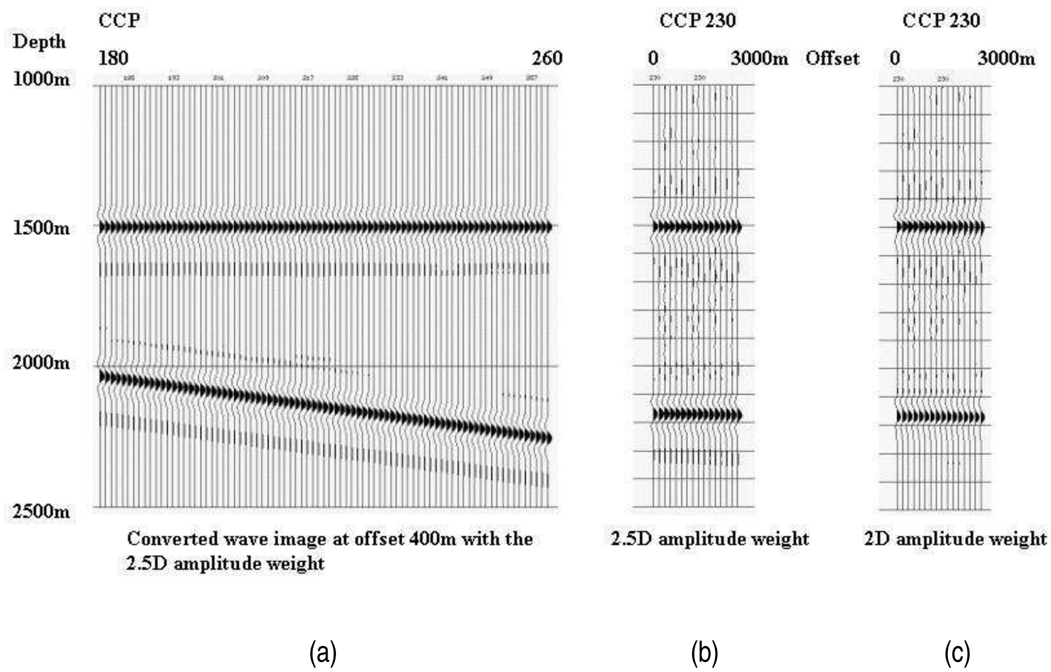


Figure 3. Amplitude comparison for 2D and 2.5D converted wave migration weight functions. (a) 2.5D weight migrated section at offset 400m with two event, one flat and one with 15 degree dip, (b) common image gather at ccp 230 using 2.5D amplitude weight, (c) common image gather at ccp 230 using 2D amplitude weight.

Conclusions

We have presented true amplitude weight functions for converted wave common offset prestack Kirchhoff migration. In our implementation, the simplified versions demonstrate negligible computational cost over that of conventional migration. The numerical modeling tests have shown well controlled amplitude behavior that successfully recovers amplitudes along depth and offset for both 2.5D and 3D cases.

Acknowledgements

Special thanks to Professor Norm Bleistein, Colorado School of Mines, for his thoughtful advice during the course of this project. Many thanks to Scott Cheadle and Daoliu Wang of Veritas GeoServices Ltd. for their technical input.

References

Bleistein, N.., 1987, On the imaging of replectors in the earth, Geophysics, 52, 931-942.

Bleisteins, N., Cohen, J. K., and Stockwell, J. W., 2001, Mathematics of multidimensional seismic inversion: Springer publications.

Dellinger, J., Gray, S. H., Murphy, G.E., and Etgen, J. T. 2000, Efficient two and one half dimensional true amplitude migration: Geophysics, 63.

Hanitzsch, C., 1997, Comparison of weights in prestack amplitude preserving Kirchhoff depth migration: Geophysics, 62, no. 6, 1812-1816.

Jin, S., 2002, Tying PS to PP depth section: Two examples of anisotropic prestack depth image of 4C data. 72nd Annual internat. Mtg., Soc. Explx. Geophysics, Expanded abstract.

Schleicher, J., Tygel, M., and Hubral, P., 1993, 3D true amplitude finite-offset migration: Geophysics, 58, no. 8, 1112-1126.

Zhang, Y., Gray, S., and Young J., 2000, Exact and approximate weights for Kirchhoff migration. 70th Annual internat. Mtg., Soc. Explx. Geophysics, Expanded abstract.