

Seismic Modelling: An Essential Interpreter's Tool

Gary F. Margrave and Peter M. Manning, CREWES, The University of Calgary, Calgary



2004 CSEG National Convention

Summary

In this tutorial presentation, we examine the common methods for simulation of seismic wave propagation including: ray tracing, convolutional modeling, the reflectivity method, 1-D theoretical seismograms, finite difference simulation, the pseudo spectral method, the finite element method, Gaussian beam modelling, and Kirchhoff modelling. We present a brief description of each method emphasizing its suitability to the needs of the seismic interpreter. In our oral presentation, we will include many examples, with both still pictures and movies. We invite the interested reader to check our website (www.crewes.org) for an updated version of this document.

Wave Theories

There are many methods which may be used to simulate how seismic waves will propagate through the real earth, and then affect the sensors which will be used to record them. The nature of seismic wave propagation can be very complex, and the various methods use compromises of various types in order to make the problem tractable. In general, modeling seismic waves requires the adoption of a particular theory of wave propagation and an analysis of the corresponding wave equation. There are a number of such theories; and, for each theory, a number of possible wave equations.

Perhaps the simplest wave theory is that of scalar waves. Also called acoustic-wave theory, this concept assumes that the physical quantity that propagates as waves can be represented by a single number, or scalar, at each location in space and each instant in time. A good example of this is the propagation of pressure waves in water. Pressure is a scalar quantity that varies with space and time. Letting P stand for pressure and assuming that the particle displacements are small compared with the wavelengths, then the so-called scalar wave equation,

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) P(x, y, z, t) = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} P(x, y, z, t), \quad (1)$$

describes the propagation of pressure waves. Equation (1) is the most commonly cited example of a scalar wave equation but there are many other possibilities that take somewhat different forms; however, this is a detail of minor importance. The common characteristic of a wave equation is that it links the second spatial derivatives of the wavefield (i.e. the pressure in this case) to the second time derivatives. Equations like (1) always arise from the fundamental physics of the problem under the assumption of small particle displacements. Essentially, two physical laws are involved, (1) Newton's second law (e.g. $f=ma$) and (2) a *constitutive* relation, such as Hooke's Law, that relates applied stress to deformation. For several centuries, mathematicians, physicists, and geophysicists have been deriving such equations and using them to describe a wide variety of wave phenomena. The symbol v appearing in front of the time derivatives is the *wavespeed* that, in this case, is the square root of the ratio of bulk modulus to density.

More complex wave theories arise when elastic media are considered. The elastic wave equation, though not shown here, can be found in fundamental texts (e.g. Aki and Richards (1985)). It differs from equation (1) primarily in that the wavefield is a vector (usually particle displacement) not a scalar, and more material parameters are required to describe the medium. The simplest elastic media require at least two material constants for their description and these are often taken to be the wavespeeds for compressional and shear waves.

More complex yet are wave theories that allow dissipation. Of course, real seismic waves always propagate with some energy loss so its simulation is of great interest. In geophysics, such lossy wave theories are often called "Q theories" after the material parameter that characterizes energy loss, Q . Though mathematically complex, enough is known about these theories to shed some significant light upon the deconvolution process. For example, Futterman (1962) showed that any linear, causal Q theory must be associated with minimum-phase dispersion. That is, we expect minimum-phase wavelets.

Ray Theories

All wave theories have a corresponding ray theory. The latter are approximate theories, usually applicable to "high" frequencies, that track the flow of wave energy along ray paths rather than wave fronts. Central tasks in ray theory are the computation of raypath geometry, the traveltimes along a raypath, and the expected amplitude behavior along the raypath. Though approximate, ray theories are popular because they are fundamentally *local* theories. In principle, to predict the wave amplitude at a particular point in space and a future time, wave theories require the contributions of all parts of the present wavefield that are *causally connected* to the particular point to be calculated. In contrast, ray theory asserts that most of these calculations will contribute little to the final result and seeks only to calculate the particularly important contributions. The latter turn out to be from points lying on raypaths that pass through the particular point. While this gives great efficiencies, it can also be very wrong if the approximations of ray theory are

violated. It can be very difficult to know when a particular result is correct or what additional effects might be missing from the calculation.

Huygens' Principle

Developed by Christian Huygens in the late 17th century, this is the idea that the wavefront at time $t + \Delta t$ is predictable from the wavefront at time t , by considering each point on the known wavefront as a source for a *secondary wavefront* or Huygens' wavelet. The Huygens' wavelet is always imagined to expand at constant velocity using the velocity at the source point. This is an excellent approximation if we take a sufficiently small time step. The future wavefront is constructed from the superposition of all possible Huygens' wavelets.

This very intuitive idea has withstood the test of time and, in the 19th century was fully justified mathematically by George Green. Huygens' principle is often used in seismic modeling.

The Convolutional Model

The simplest, useful model of a seismic trace is that it is the convolution of the wavelet and the reflectivity. This begs the questions: "What do we mean by wavelet?" and "What do we mean by reflectivity?". The obvious answer to the first is that the wavelet is the temporal signature emitted by the source, or *source waveform*. The *reflectivity* also has a standard meaning within exploration communities as the time series created by placing the normal-incidence reflection coefficients at their two-way traveltimes. Seeking a justification within mathematical physics for this model leads to Green's Theorem which, in the present context, states that the seismic trace is the convolution of the source waveform with the *impulse response of the earth*. By the latter, we mean the response of the earth to a perfectly impulsive source (called a Dirac delta function in theoretical physics). The impulse response contains the reflectivity as a subset but it has far more including all multiples, converted waves, and attenuation effects. Thus the popular convolutional model seems to be missing some physics; however, it is still useful in several contexts. One is if we wish to build a seismogram to compare to final seismic images. In this case, we have suppressed multiples and corrected for attenuation and effectively converted the impulse response into the reflectivity. Another useful application, is to consider that the convolutional model applies in a small time window where by wavelet we mean the source signature after modification by attenuation and multiples from above the window. Finally, we emphasize that, given a result from any modelling program from a source that is nearly impulsive in space or time, the result for a more spatially distributed or temporally elongated source can be obtained through convolution with the new source configuration. The modelling software need not be rerun.

Time Stepping

Many, though not all, modelling methods proceed through a technique called time stepping. If we consider a wavefield in n spatial dimensions to be an $n+1$ dimensional object where the extra dimension is time, then by *snapshot* we will refer to a slice through this wave object at constant time. Time stepping is the process of calculating a wavefield snapshot at some future time given snapshots of the present and a few past times. Most commonly, time stepping is used in finite difference modelling but it is also used in raytracing and Gaussian beam methods.

1-D Synthetic Seismograms

This is usually the first type of modelling done in an area where wells have been drilled and sonic logs have been run. The idea is at least 50 years old and is often credited to Goupillaud. Essentially, the 1D wave problem has an almost complete solution that includes all possible multiples. Goupillaud suggested that a layer model might be constructed from well logs such that all layers have an identical two-way traveltime that is equal to the desired sample rate of the seismogram. Then any primary or multiple will fall exactly upon a particular sample. Later algorithms dispensed with the need for equal traveltime sampling but were still able to calculate all possible multiples. It was through study of this algorithm that it was realized that the multiple train through a stratigraphic sequence can easily overwhelm the primaries. The calculation uses great detail in the z (vertical) direction, but assumes that there is no variation laterally. Standard methods are available to translate the well log data into a form that the programs may use. The main decision that must be made is of the bandwidth and phase of filters that might best simulate the recorded seismic data. An example is shown in Figure 1.

Advantages

- Few modelling choices and therefore most objective
- Great detail in the z -direction
- All possible multiples can be included
- Attenuation is easily included

Disadvantages

- Assumes 'layer cake' geology
- Simulates only a zero offset stacked trace at one position
- Multiple calculation depends upon length of log

Variations

- Intelligent interpolation between adjacent wells can simulate a stacked section.
- An offset section may be obtained with additional computer effort. Low velocity zones may have to be edited out.

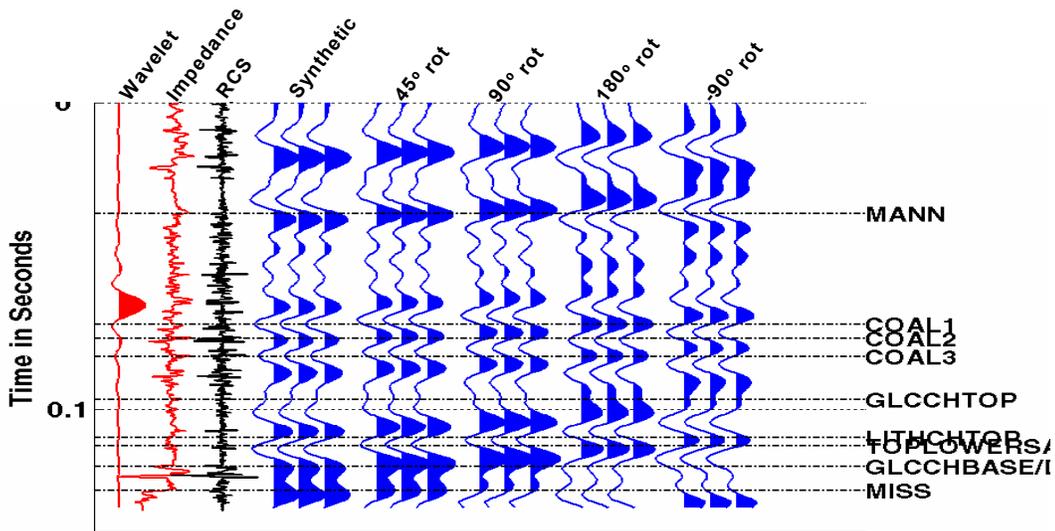


Figure 1: A normal incidence synthetic seismogram calculated from well log input

Ray tracing

This is usually done through a model of the earth which is divided into blocks of relatively constant velocity. The raypaths obey Snell's law at block boundaries, and so make realistic bends and record realistic amplitude changes as they cross these boundaries. The amount of raypath spreading may be used to define part of the energy amplitude changes. A great difficulty with ray theory is knowing when and how to calculate a sufficiently dense set of rays to accurately represent a model. What might initially seem as a very dense set of rays may develop "holes" or gaps in coverage as the rays diverge and converge in their propagation. Additionally, raytracing is almost always iterative because there is no known way to calculate directly the raypath between a specific source and receiver in an arbitrarily complex medium. Instead, one "shoots" a fan of rays out from a source and finds two that bracket the receiver. Then a new fan is calculated with take-off angles lying between the two bracketing rays. This process is repeated until the process converges in the sense that a ray is found to come within a specified *capture radius* of the receiver. A popular application of raytracing is the iterative calculation of multi-offset records, which later may be processed and stacked using standard processing software (Figure 2). Such gathers can easily be made to simulate elastic data if reflection and transmission amplitudes are calculated with the Zoeppritz equations (Figure 3).

Advantages

- Any style of geologic section may be modelled
- Zero offset (stacked) sections may be efficiently modelled.
- Shot records or gathers may be simulated.

Disadvantages

- 'Blocking' which preserves the most important features of the geological section may be difficult to do.
- Inappropriate 'blocking' may cause significant misleading features which are not easily detected.
- Block boundaries which are meant to be continuous must be represented by a large number of points in order to prevent erratic raytracing behaviour. The procedure to interpolate these points may cause problems, especially where there are also real discontinuities (faults).
- It is difficult to simulate diffractions. Raytraced models without diffractions are highly artificial and do not migrate well.

Variations

- Rays may be given realistic curvature within blocks by specifying velocity gradients within the blocks. This may be especially useful for modelling refraction events.

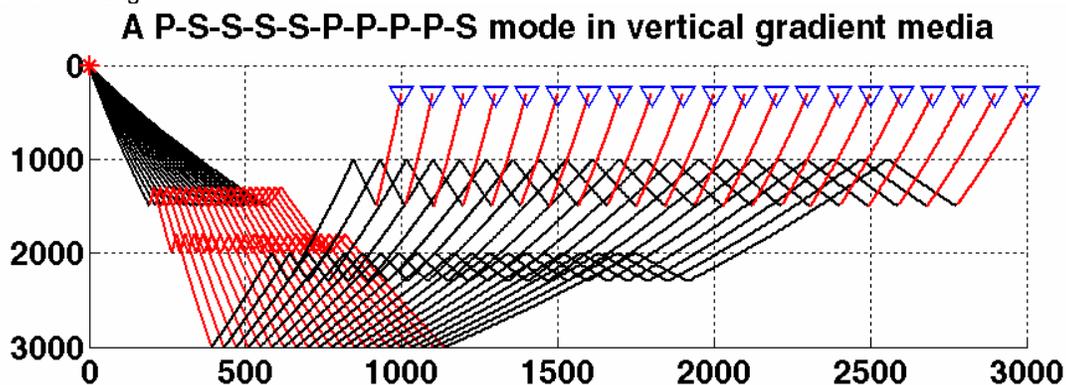


Figure 2: Ray tracing in horizontally layered media.

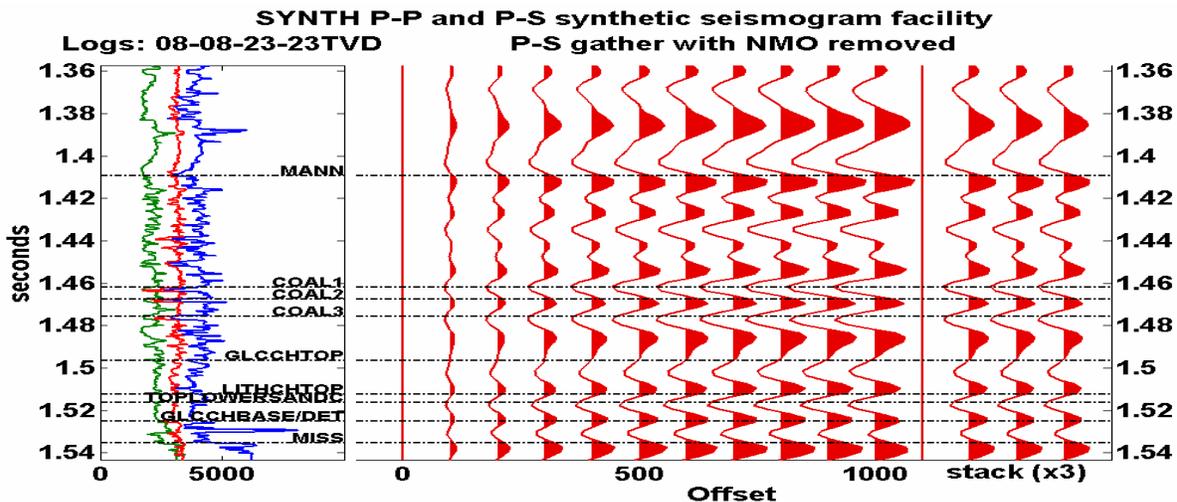


Figure 3: A P-S offset response calculated with raytracing

Gaussian beams

This is similar to the ray tracing method except that the rays are simulated with a finite width. The ray width is typically a Gaussian shape, hence the name. Amplitude variation across the beam is calculated with a local approximation to the wave equation. A beam is affected by the entire range of material properties that it encounters, and so are not as vulnerable to small anomalies or discontinuities in the slopes within the geologic model. A great advantage of the Gaussian beam method is that it is far easier (than with raytracing) to obtain a set of beams that fully covers a model. Conceptually Gaussian beams lie midway between rays and waves.

Advantages

- More forgiving than raytracing of an insufficiently smoothed geologic model.
- Better approximation to the direction in which real sound propagates in the vicinity of real discontinuities.

Disadvantages

- More complex programming.
- Difficult to assess the accuracy of a simulation.

Hyperbolic superposition

Even the most sophisticated raypath modelling is a less accurate representation of the details of a wavefront than a direct wavefront model. The more notable features of these models are the 'annealing' of gaps caused by reflectivity discontinuities, and the energy spread into diffractions at the ends of reflection events.

The most elementary method using wavefronts derives from Huygens' principle and involves the superposition of many elementary waves. Since the surface recording by the geophones of a spherical wavefront has the shape of a hyperbola, the method is often formulated as the superposition of hyperbolae. Conceptually, a single hyperbola represents the energy scattered from a single point in the subsurface and the method is the summation of many such scatterpoint responses. The hyperbola shape is determined by the rms velocity while its absolute time is determined by the average velocity. The response of a horizon is then simulated as a sum of responses from the single points making up the horizon. This method is especially convenient for simulation stacked sections where there are not too many velocity anomalies, and so the hyperbola shape is predictable (Figure 4).

Advantages

- An economic and robust means of wavefront modelling.
- Shows the shape of diffraction curves that will appear on real seismic sections.
- Will migrate nicely

Disadvantages

- Will give accurate results only in models with very simple velocity structures.
- Summations into a continuous reflection is often quite imperfect, dependent on how the horizon is sampled.

Finite differencing

The finite difference method may be used to model sonic wave propagation through a geologic model with very few of the problems mentioned above, but a completely different set of problems appear. The new problems are associated with numerical analysis, involving dispersion and edge effects.

In this method, the propagating waveform is specified from its starting point as a complete wave, and the geologic model is specified throughout by its density and compressibility (acoustic case, Figure 5), or with the additional specification of shear strength (elastic case, Figure 6). The waves then propagate very naturally in response to the laws of physics (the wave equation).

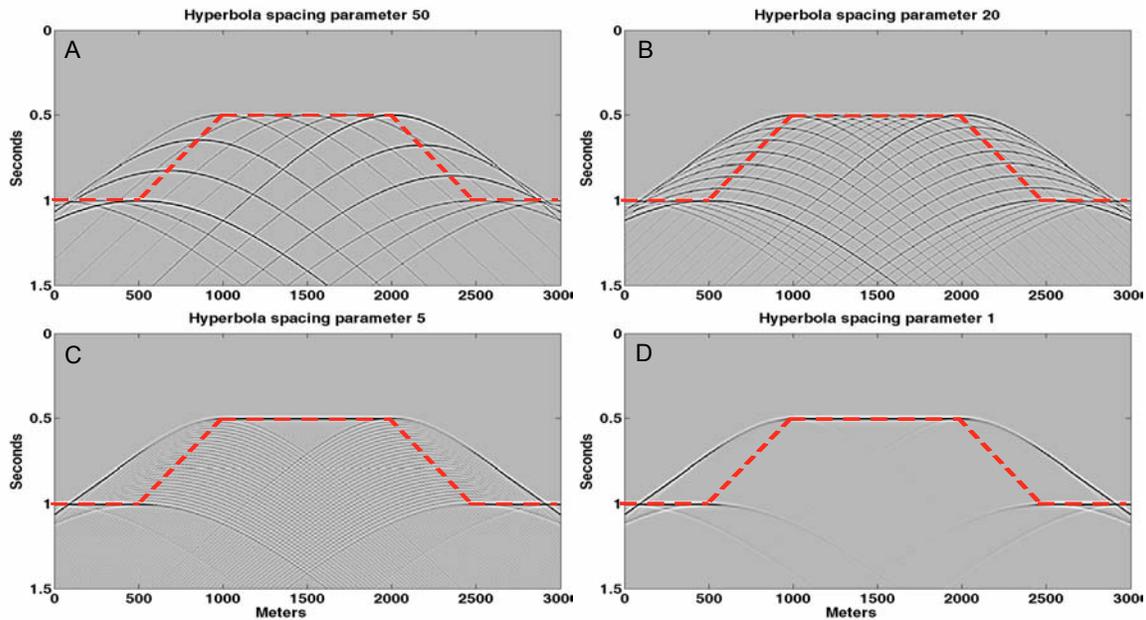


Figure 4. Panels A-D illustrate the progressive emergence of the image of an idealized reef structure (red dashed line) as the number of hyperbolae becomes ever greater. The final image (D) is an accurate depiction of the zero-offset response of this structure for constant velocity.

Advantages

- The method shows subtle phase changes without any extra effort. These may be caused (for example) by interactions at arbitrary angles with closely spaced reflectors or the free surface.
- The method also shows waveform edge effects without extra effort. These may include waveform annealing across gaps, or diffraction curves from discontinuous reflectors.

Disadvantages

- A certain amount of numerical dispersion is inherent with the process, resulting in distorted waveforms. A special effort and long computer times are often required to deal with it.
- The process treats model edges as physical boundaries, causing reflections that can mask the desired simulation. Special effort is often required to reduce these effects also.
- Models with a large range of velocities may cause problems. Areas with much lower velocities (e.g. half of the maximum) will have events with much more dispersion.

Variations

- The acoustic model is theoretically valid only for liquids, but is sufficiently accurate for many general applications.
- The elastic model requires more computer effort, but may be necessary for the more subtle studies.

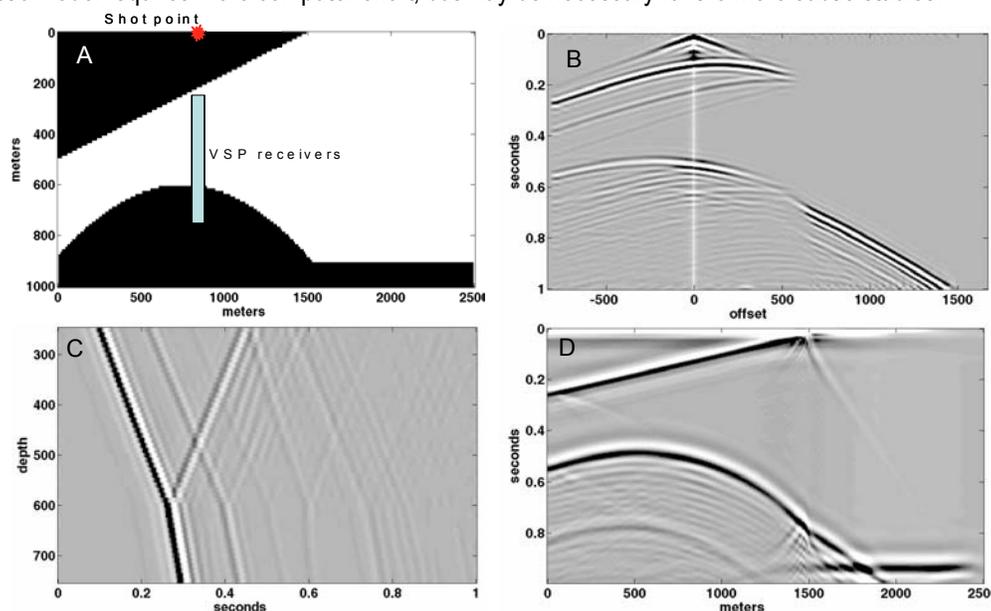


Figure 5. (A) A velocity model where black is 4000 m/s and white is 2000 m/s. (B) An acoustic shot record simulated by finite differencing. (C) A VSP simulated by Acoustic finite differencing. (D) A zero-offset simulation (exploding reflector).

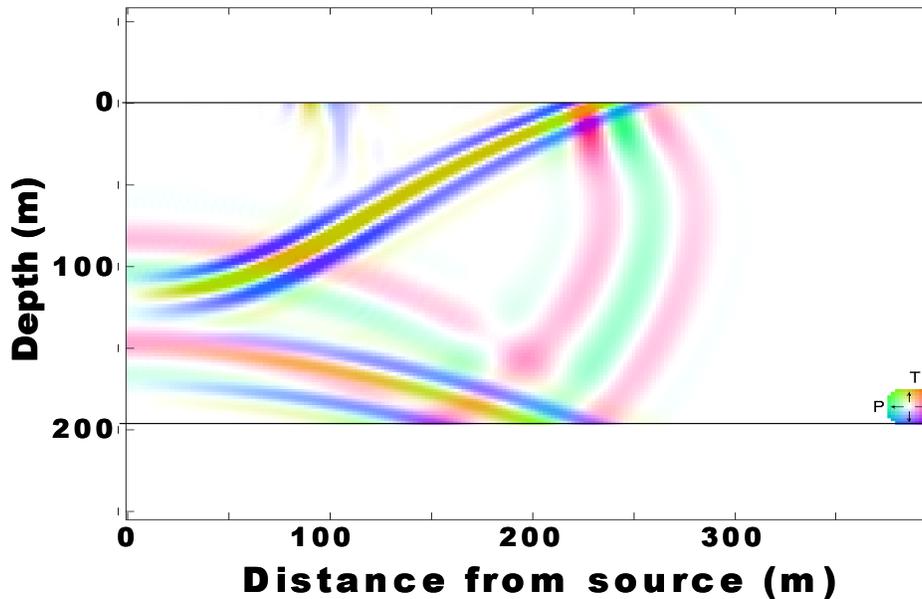


Figure 6. A snapshot of an elastic wavefield showing a surface wave, a P and S reflections, and up and downgoing waves. Red-cyan-red is a P wave while blue-green-blue is an S wave.

Pseudo Spectral Methods

A major problem with the finite-difference method is that simple finite-difference operators require many samples per wavelength in order to control artefacts such as grid dispersion. One way to improve this is to go to more sophisticated finite-difference operators. Yet another alternative is the pseudo spectral method where the spatial derivatives are calculated in the wavenumber domain. This gives theoretically optimum performance requiring only two samples per wavelength but at the price of Fourier wrap-around artefacts. The method proceeds by time-stepping in exactly the same fashion as the finite difference technique.

Finite Element Methods

Rather than employ an approximate derivative estimate at each point in a grid, the finite-element method breaks a complex body into a finite number of polygonal regions and solves the wave-equation exactly within each region. Though capable of great accuracy, this method requires very sophisticated model-building software.

The Reflectivity Method

First proposed by Fuchs and Mueller (1971) this is a sophisticated technique for the creation of the complete elastic, body-wave response from a horizontally layered system. It was later extended by Kennett (1983) to include the theory of a generalized reflection and transmission from layered system. These generalized responses include all possible multiples, mode conversions, and transmission losses.

Conclusions

A great many tools are available for seismic modelling. There is no single best method that suits for all purposes. Instead the seismic interpreter must be aware of the range of methods and their strengths and weaknesses in order to make the best choice for the purpose at hand.

References

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