Ray-Path Elastic Impedance

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Abstract

A new ray-path elastic parameter is derived by extending the concept of elastic impedance (EI), We propose the so-called ray-path elastic impedance(REI). REI includes not only acoustic parameters (i.e. P-wave velocity and density), but also S-wave velocity and other lithological parameters and thus is advantageous for interpretation. Similarly to acoustic impedance and EI, REI represents the P-wave reflectivity in in recursive formula that is suitable to the inversion of seismic data. REI includes not only AI (i.e. P-wave velocity and density), but also S-wave velocity and other lithological parameters. AVO attributes, including the intercept, slope, angle and ray-parameter stacks are derived in a way similar to EI. However, unlike the traditional EI, REI has direct physical meaning of impedance to a wavelet propagating at oblique incidence through the medium and can be obtained by a τ-*p* transformation of CMP gathers. In addition, REI does not require normalization for comparison with the acoustic impedance. By combining REI inversions at different ray parameters, we can obtain additional and reliable lithological information.

Introduction

Conventional seismic trace inversion is based on an assumption that the P-wave strikes subsurface interfaces normally (Lindseth, 1979; Cooke and Schneider 1983). When the offset range of a CDP gather is small and the reservoir is very deep, such assumption is approximately satisfied and then this trace inversion method can produce a reliable result. However, when amplitude variation versus offset (AVO) effect exists in the CDP gather, especially in the large offset range, the actual reflection coefficient will be different significantly from that of a normal incidence. In such case, it is necessary to modify the conventional seismic trace inversion method by taking the AVO effect into consideration.

Connolly (1999) put forward the concept of elastic impedance (EI), which takes account the effect of *P*-wave reflectivity versus incident angle in order to solve the post-stack seismic trace inversion problem with large offset. When EI function is used in seismic trace inversion, the ratio of S-wave to P-wave velocities, $\gamma = \beta / \alpha$, is assumed to be constant, and the seismic data are replaced with a common angle stack or an AVA fitting stack. Furthermore, EI must be normalized and it is not easy to control (D.N.Whitcombe, 2002). This restricts the applicability of EI.

Below, we propose a new elastic parameter that we call the ray-path elastic impedance (REI). Similarly to EI, ray-path reflectivity is given by a recursive formula and is suitable for the inversion of large-offset and VSP data. One critical advantage of this approach is that REI relates to the reflectivity information gathered along an actual ray-path rather then from non-physical AVA stacks used in EI. Ray-path reflectivity could be obtained directly from slant-stacked CMP gathers and allowing extraction of REI attributes without reliance on a linear amplitude versus angle relation. In addition, the β / α = const requirement of EI is relaxed to a more general approximation $\rho \sim \beta^k$ for the density-*S*-wave velocity relationship. By means of REI inversion, we can obtain more reliable lithological information about fluids, porosity, sand-to-mud ratio *etc*. It is helpful to decrease non-uniqueness in the conventional seismic trace inversion and thus to improve the accuracy of reservoir prediction.

Ray-path Elastic Impedance and Elastic Impedance

For small difference in *P*-wave velocities, *S*-wave velocities and bulk densities across an interface, the elastic impedance (*EI*) is defined as (Connolly, 1999)

$$EI = \rho^{1-4K\sin^2\theta} \alpha^{1+\tan^2\theta} \beta^{-8K\sin^2\theta}, \tag{1}$$

where α is *P*-wave velocity, β is *S*-wave velocity, γ is density, θ is the *P*-wave incidence angle,

$$K = \frac{1}{2}\left[\left(\frac{\beta_{i+1}}{\alpha_{i+1}}\right)^2 + \left(\frac{\beta_i}{\alpha_i}\right)^2\right] = \gamma^2, \tag{2}$$

and *i* is the layer index. If *EI* is known, the reflectivity of the *i*th layer can be written as

$$R_{pp_i}(\theta) = \frac{EI_{i+1} - EI_i}{EI_{i+1} + EI_i} \approx \frac{1}{2}\ln\frac{EI_{i+1}}{EI_i} = \frac{1}{2}\ln\frac{\rho_{i+1}^{1-4K\sin^2\theta}\alpha_{i+1}^{1+\tan^2\theta}\beta_{i+1}^{-8K\sin^2\theta}}{\rho_i^{1-4K\sin^2\theta}\alpha_i^{1+\tan^2\theta}\beta_i^{-8K\sin^2\theta}}, \tag{3}$$

Note that the recursive equation (3) is based on two approximations:

(1) Parameter *K* must be constant; and

(2) The *P*-wave incident angle θ must be constant for all reflection interfaces.

Following the ray-parameter rather then incident-angle parameterization, the Zoeppritz equation can be approximated in terms of elasticity modulus as (Wang, 1999)

$$R(p) \approx R_f(p) - \frac{2\Delta\mu}{\rho}p^2, \tag{4}$$

where *p* is the ray parameter, ρ is density, μ is shear modulus, and

$$R_{f_i}(p) = \frac{1}{2}\Delta\ln\rho\alpha + \frac{1}{2}\frac{\Delta\alpha}{\alpha}\tan^2\theta = \frac{\rho_{i+1}\alpha_{i+1}\cos\theta_i - \rho_i\alpha_i\cos\theta_{i+1}}{\rho_{i+1}\alpha_{i+1}\cos\theta_i + \rho_i\alpha_i\cos\theta_{i+1}} \approx \frac{1}{2}\ln\left(\frac{\rho_{i+1}\alpha_{i+1}}{\cos\theta_{i+1}}\bigg/\frac{\rho_i\alpha_i}{\cos\theta_i}\right) \tag{5}$$

is the reflectivity when a plane wave strikes the fluid interface, and θ is P-wave incident angle. In equation (4), the term $R_f(\theta)$ related to the *P*-wave properties is already recursive (eq. 5). In order to represent *R(p)* in recursive form, we need to cast its second term in recursive form as well.

According to Snell's law, we have the following two approximations (Aki and Richards 1980; Wang 1999):

$$\Delta\theta \approx \frac{\Delta\alpha}{\alpha}\tan\theta, \qquad \Delta\varphi \approx \frac{\Delta\beta}{\beta}\tan\varphi, \tag{6}$$

where φ is the *P-SV* wave reflection angle. Considering

$$\Delta\mu = \beta^2\Delta\rho + 2\rho\beta\Delta\beta, \tag{7}$$

equation (4) can be expressed as

$$R_{pp_i}(p) \approx \frac{1}{2}\ln\left(\frac{\rho_{i+1}\alpha_{i+1}}{\cos\theta_{i+1}}\bigg/\frac{\rho_i\alpha_i}{\cos\theta_i}\right) - 2\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta\beta}{\beta}\right)\sin^2\varphi. \tag{8}$$

For small *P-SV* reflection angles, equation (8) can be further approximated to

$$R_{pp_i}(p) \approx \frac{1}{2}\ln\left(\frac{\rho_{i+1}\alpha_{i+1}}{\cos\theta_{i+1}}\bigg/\frac{\rho_i\alpha_i}{\cos\theta_i}\right) - 2\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta\beta}{\beta}\right)\tan^2\varphi. \tag{9}$$

Note that in order to insure accuracy of the approximation, only the *P-SV* reflection angles need to be small but the corresponding P-wave incident angle could still be large. For example, if α/β = 2:1 and the *P-SV* reflection angle of $\varphi = 30^0$, the P-wave incidence angle will be $\theta = 90^0$. Therefore, with high α/β ratios (which is often typical), equation (9) can be used for large incident angle or long-offset seismic data.

Assuming $\Delta\rho/\rho \approx k\Delta\beta/\beta$, with *k* constant, equation (9) can be rewritten as:

$$R_{PP_i}(p) \approx R_{f_i}(p) - 2(\mathrm{k}+2)\frac{\Delta\beta}{\beta}\tan^2\varphi \approx R_{f_i}(p) - 2(k+2)\frac{\sin\varphi}{\cos\varphi}\Delta\varphi$$
$$\approx R_{f_i}(p) + 2(\mathrm{k}+2)\frac{\Delta\cos\varphi}{\cos\varphi},$$

and finally, be represented as a ratio of ray-path elastic impedances:

$$R_{PP_i}(p) \approx \frac{1}{2}\ln\left(\frac{\rho_{i+1}\alpha_{i+1}}{\cos\theta_{i+1}}\cos^{4(k+2)}\varphi_{i+1} \Big/ \frac{\rho_i\alpha_i}{\cos\theta_i}\cos^{4(k+2)}\varphi_i\right) \approx \frac{1}{2}\ln\frac{REI_{i+1}}{REI_i}, \quad (10)$$

where REI is defined as:

$$REI_i = \frac{\rho_i\alpha_i}{\cos\theta_i}\left(1-\frac{\beta_i^2}{\alpha_i^2}\sin^2\theta_i\right)^{2(k+2)} = \frac{\rho_i\alpha_i}{\sqrt{1-\alpha_i^2p^2}}.(1-\beta_i^2p^2)^{2(k+2)} \quad (11)$$

For $\Delta\rho/\rho \approx 0$, or k =0, equation (11) yields:

$$REI_i = \frac{\rho_i\alpha_i}{\cos\theta_i}\left(1-\frac{\beta_i^2}{\alpha_i^2}\sin^2\theta_i\right)^{4} = \frac{\rho_i\alpha_i}{\sqrt{1-\alpha_i^2p^2}}.(1-\beta_i^2p^2)^{4} \quad (12)$$

The meaning of REI defined here is similar to that of EI but can be used for ray-path seismic trace inversion. When p = 0, REI becomes the acoustic impedance (*AI*) and equation (10) becomes the reflectivity of normal incidence. All the modeling and inversion methods based on AI or EI are applicable to REI as well.

Because in the construction of REI logs we do not assume a constant incidence angle on all the interfaces, the resulting variation of REI due to varying offsets is typically smaller compared to EI (eq. 1). This occurs because for the same ray path, the lower-velocity (with often higher α/β) layers have smaller incident angles and consequently lower AVA effects. As a result, and unlike the EI function (Whitcombe, 2002), REI logs do not require normalization when compared to AI logs.

Note that REI (eq. 4) is explicitly separated into the fluid-fluid and rigidity terms. The rigidity term in REI is only related to the shear wave velocity and constant ray parameter p, and thus it is the same for hydrocarbon and water-saturated reservoirs. Because REI reflects the pore-fluid rock properties better, it might be useful in the prediction of the fluid state of reservoir and in time-lapse seismic observations.

The Accuracy of the EI and REI approximations

To verify the accuracy of REI equation, we used a three-class standard model (Rutherford, 1989; Table 1) and compute the dependences of P-wave reflectivity on the incident angle using the exact Zoeppritz equation, EI equation (1), and REI equation (12). In our experience, sedimentary rock α / β ratio could range from 1.5 to 4 or even higher in unconsolidated rocks; and consequently we try multiple *K* values in these calculations.

Figs.1-3 show three classes of gas-sand-bottom and top AVO responses. In most cases, the REI equation appears to be closer to the exact solution than EI. Note that, for the EI equation, the first class gas sand AVO responses could be interpreted as the second class, or vice versa if wrong K values were used. This means that it will lead to wrong prediction of potential hydrocarbon reservoirs or other reservoir properties in EI inversion.

Class	Rock	α (m/s)	β (m/s)	ρ (g/cm^3)
I	Gas sand	2438	1625	2.16
	Shale	2700	1825	2.25
II	Gas sand	2438	1025	2.14
	Shale	2480	1232	2.16
III	Gas sand	3048	1244	2.40
	Shale	2838	1650	2.28

Table 1 –Standard elastic models in computations

Conclusions

The ray-path elastic impedance (REI) is able to reproduce accurate oblique incidence reflection coefficient series from a simple approximation similar to that of the acoustic impedance. By removing the limitation of a constant *P*- to *S*-wave velocity ratio assumed by the EI formula, REI approximation becomes significantly more general and accurate.

REI has a simple and explicit physical meaning of impedance to a plane ray wave propagating through the medium. Using the REI concept, we can solve the nonzero-offset seismic trace inversion problem and reduce the non-uniqueness of the traditional impedance inversion. By contrast to the traditional seismic trace inversion, REI method would be particularly suited to the inversion of nonzero-offset VSP data. In addition, REI does not require normalization that is usually necessary with EI inversion.

Ray-path formulation opens a number of venues for application of EI in seismic interpretation and inversion. Because of its relation to the ray parameter, REI can be derived directly from τ-*p* stacks either using or bypassing the AVA stacks. This could make joint inversion of travel times and amplitudes possible. By considering different ray paths and azimuths, REI might also be helpful in discrimination of anisotropy properties of reservoirs.

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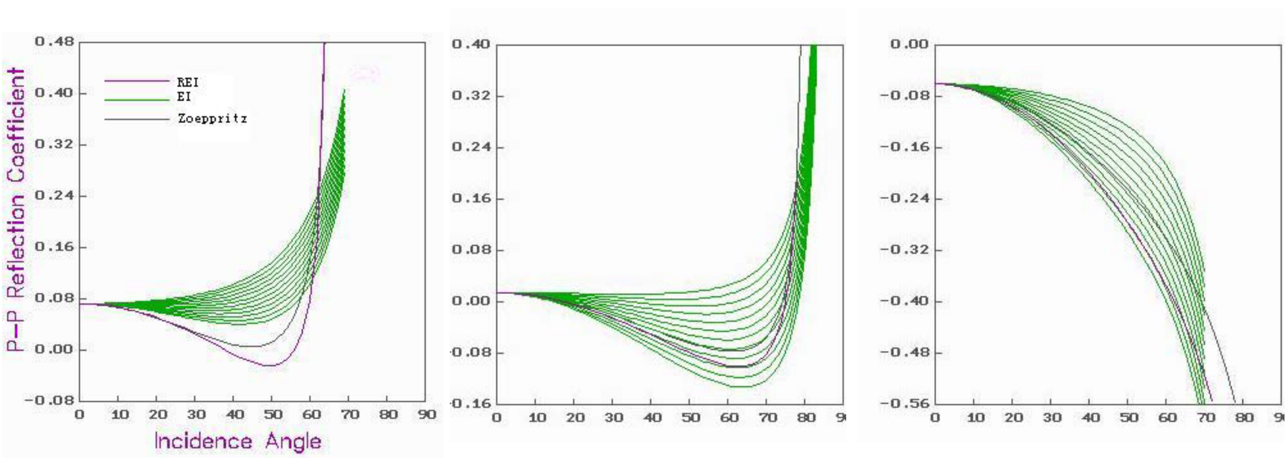

Fig.1 First class gas sand AVO response. Fig.2 Second class gas sand AVO response. Fig.3 Third class gas sand AVO response. Purple lines represent the results calculated from the REI equation (12), black lines are from the exact Zoeppritz equation, and green lines are from EI equation (1) with varying values of K. Reflection coefficients corresponding to the gas sand/shale interface models.