Curvelet processing and imaging: 4D adaptive subtraction

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Abstract

With burgeoning world demand and a limited rate of discovery of new reserves, there is increasing impetus upon the industry to optimize recovery from already existing fields. 4D, or time-lapse, seismic imaging holds great promise to better monitor and optimise reservoir production. The basic idea behind 4D seismic is that when multiple 3D surveys are acquired at separate calendar times over a producing field, the reservoir geology will not change from survey to survey but the state of the reservoir fluids will change. Thus, taking the difference between two 3D surveys should remove the static geologic contribution to the data and isolate the time-varying fluid flow component. However, a major challenge in 4D seismic is that acquisition and processing differences between 3D surveys often overshadow the changes caused by fluid flow. This problem is compounded when 4D effects are sought to be derived from legacy 3D data sets that were not originally acquired with 4D in mind. The goal of this study is to remove the acquisition and imaging artefacts from a 4D seismic difference cube using Curvelet processing techniques.

The denoising problem

In this paper, we argue that computing 4D difference cubes can be recast into the framework of solving a generic denoising problem that estimates the model **m** from noisy data

$$\mathbf{d} = \mathbf{m} + \mathbf{n} \tag{1}$$

with Gaussian noise **n**. The solution of this inverse problem can be written in terms of the following variational problem

$$\hat{\mathbf{m}}: \ \min_{\mathbf{m}} \frac{1}{2}\left\|\mathbf{d}-\mathbf{m}\right\|_2^2 + \mu J(\mathbf{m}) \tag{2}$$

where $J(\mathbf{m})$ is an additional penalty function that contains prior information about the model, such as particular sparseness constraints [10,11]. The control-parameter μ rules how much emphasis one would like to give to the prior information on the model.

The question now is: how can we solve this denoising problem effectively? In other words, how can we construct a diagonal decision operator that minimizes the energy difference between the estimate and the true model given the prior information? It appears from the work of [7], [10] and others that, for a certain class of models, one can obtain nearly optimal denoising results, i.e. near optimal SNR for denoised data, by projecting noisy data onto a basis-function representation that is optimal for that particular class of models. In doing so, most of the model's energy will reside in only a few coefficients, allowing for the definition of a shrinkage estimator that separates noise from the model. For basis functions that are also local, one can show that soft thresholding of the coefficients suffices to approximately solve the above denoising problem, i.e.

$$\hat{\mathbf{m}} = D(\mathbf{d}) = \mathbf{B}^{-1}\theta_{\mu}(\mathbf{Bd}) \tag{3}$$

where **B** is the basis-function expansion and $\mathbf{B}^{-1}$ is its (pseudo)-inverse. θ_μ is a thresholding operator with a threshold that, for orthonormal basis functions, equals $\mu=\sigma(2\log_e N)^{1/2}$ with σ the standard deviation of the noise and N the number of data [10,7]. Soft thresholding solves Eq. (3) on the coefficients for a penalty function given by the L^1-norm, i.e. $J(\mathbf{m}) = ||\mathbf{m}||_1$. Comparison of equations 2 and 3 establishes the connection between the threshold and the control parameter μ.

Wavelets, and their recent extension to Curvelets, derive their success mainly from their ability to represent a model locally and sparsely. This facilitates the definition of simple non-linear thresholding estimators that locally decide (adaptively filter) whether a certain event pertains to the model or to the noise. The question now is: can we extend these Wavelet-adaptive filtering ideas to the computation of 4D difference cubes? Before answering this question, let's be more specific with respect to the choice of the appropriate basis functions for representing seismic data volumes.

Localized basis-function decomposition

Curvelets, as proposed by [2,1,4], constitute a relatively new family of non-separable Wavelet bases that are designed to effectively represent seismic data with reflectors that generally tend to lie on piece-wise smooth curves. This property makes Curvelets suitable to represent reflectors within vertical/horizontal slices of migrated volumes- including faults. For these type of signals, Curvelets obtain nearly optimal sparseness, yielding (i) a rapid decay in reconstruction error as a function of the largest coefficients; (ii) concentration of the signal's energy in a limited number of coefficients; and (iii) relatively easy separation of noise versus model. So how do Curvelets obtain such a high non-linear approximation rate? Without being all inclusive [see for details 2,1,4,6], the answer to this question lies in the fact that Curvelets are

- *multi-scale*, i.e. they live in different dyadic corona (see Fig. 1 (Left) in the (k_x, k_y)-domain, or, equivalently, the (k_x, k_z)-domain)
- *multi-directional*, i.e. they live on wedges within these corona (see Fig. 1 (Left))
- *anisotropic*, i.e. they obey the scaling law $width \propto length^2$
- *directional selective*, with # $orientations \propto length^2$
- *local*, both in (x,y) and (k_x, k_y)
- almost *orthogonal*- they are tight frames with a moderate redundancy.

These properties make Curvelets the appropriate basis functions for dealing with seismic data. However, so far no 3-dimensional Curvelets have been developed. As a result, we have the choice to apply Curvelet transforms to either depth slices or to the in-line direction, followed by applying a shift-invariant Wavelet transform [5] in the depth or cross-line direction.

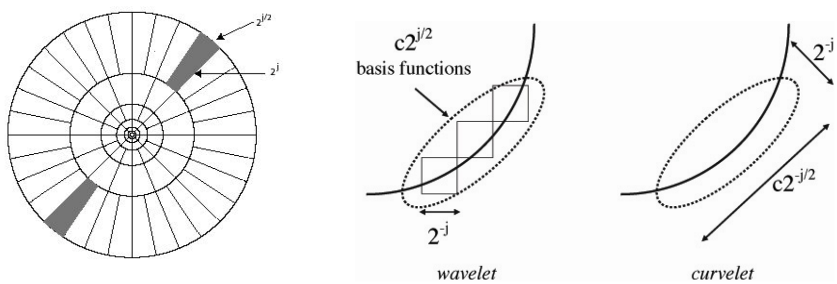

*Figure 1. **Left**: Curvelet Partitioning of the Frequency Plane [modified from 3]*
***Right**: Comparison of non-linear approximation rates for Curvelets and Wavelets [modified from 6]*

Adaptive subtraction

Given the above basis-function decomposition, how can we recast the computation of 4D difference cubes into a denoising problem? The problem is that we would like to reduce *difference-noise*, which we define as representing (i) incoherent noise in the two volumes; (ii) misalignments of reflectors between the two surveys; and (iii) differences in acquisition footprints. During this noise removal, we would like to preserve and emphasize 4D effects. In light of these objectives, we choose not to actually subtract the two datasets, but rather filter one with the other. We consider one dataset to be the "noise" $\mathbf{n}$ of Eq. 1 (not to be mistaken with the *difference-noise* defined above) of the other, the data $\mathbf{d}$, and carry out our program by a denoising procedure- yielding an estimate for the *difference-signal* $\hat{\mathbf{m}}$.

The crux of any filtering operation lies in being able to optimally represent both the model $\hat{\mathbf{m}}$ (the *difference-signal*) and "noise" $\mathbf{n}$ (the second dataset). This optimality refers to the basis-function's ability to sparsely represent both the model and the noise. If, in addition, the basis functions are localized, *superior* filtering results are obtained, i.e. better signal-to-noise ratios. This superiority is not only due to a reduction of the dimensionality of the subtraction problem but is also related to the locality of the basis functions, permitting the use of non-linear estimators based on thresholding [see e.g. 10]. These estimators make localized decisions on whether certain events are 4D effects or belong to the *difference-noise*, based on the magnitude of the coefficients in the basis-function decomposition. The better the basis functions approximate the model and, in the case of coherent noise, the noise, the better the local decision operator will be able to discriminate between *difference-signal* and *difference-noise*. Since the basis functions we are using are local, the decision operator can be a simple hard- or soft-thresholding operator [10], which shrinks the coefficients towards zero that are below a certain threshold. We refer to another paper in these proceeding (by the same last author) where the same technique is used to remove predicted multiples.

As the example in the next section demonstrates, the optimal denoising capabilities for incoherent noise (cf. Eq. 3) carry over to coherent noise removal, provided we have a reasonable consistency in the imaging and data acquisition between the two vintages. By choosing a threshold defined by one of the vintages, i.e.

$$\mu \sim 3\eta|\mathbf{Bn}| \quad [4]$$

with **B** representing the combination of Curvelet and Wavelet decompositions, we are able to adaptively decide whether a certain coherent feature belongs to an actual 4D effect or to *difference-noise*. In this expression, η is a threshold control-parameter and the 3 is related to the 95% confidence interval.

Application

Two 3D seismic surveys were acquired at the Ula North Sea oil field in 1984 and 1999 respectively. Even though the datasets have been processed in parallel, the fact that the two surveys were shot in perpendicular directions and with different acquisition systems may complicate the extraction of 4D effects [9].

For the present case, our goal is to suppress the remaining imaging and acquisition artefacts such that the true 4D signature may be better expressed. The Curvelet-estimation technique presented in this paper was used and we proceeded as follows. First, the Curvelet transform was applied along slices in the in-line direction of the 1984 and 1999 vintage datasets. This was followed by a non-decimated Wavelet transform in the cross-line direction. Subsequently, the threshold was calculated according Eq. 4: **n** given by the 1984 dataset and η=1. Finally, we applied the thresholding procedure defined in Eq. 3 on **d** given by the 1999 dataset. The effects of the thresholding on the vector containing the curvelet coefficients are shown in Fig. 2.

To illustrate our results, we compare in Fig. 3 the reconstruction from the thresholded coefficients with the ordinary difference cube along the picked Top Ula horizon, which caps the producing reservoir. The superimposed logarithmic color scheme is representative for averaged amplitude difference magnitude over the first 5 samples below the picked horizon. Hot colours correspond to amplitude brightening while cold colors correspond to dimming. The top of the reservoir is located at record 200, trace 26- which is not within the range of the data we presently have available. The region shown is near a gas injector.

Figure 3A shows the difference magnitudes for the ordinary difference cube, which suffers from artefacts induced by the *difference-noise*. These artefacts overshadow the smaller differences caused by true 4D effects. Difference is particularly large at the faulted and steeply dipping part of the horizon near record 10, trace 25. This is likely a result of reflector misalignment between the two surveys. Figure 3B shows difference projected onto the Top Ula horizon after Curvelet domain adaptive subtraction. It appears that *difference-noise* has been significantly reduced and it is believed that the remaining differences mainly result from true reservoir changes.

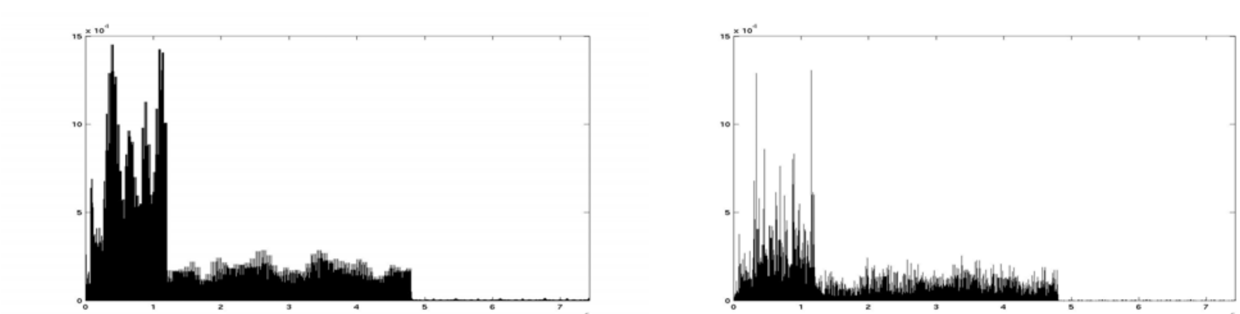

Figure 2. Curvelet coefficient vectors before and after thresholding

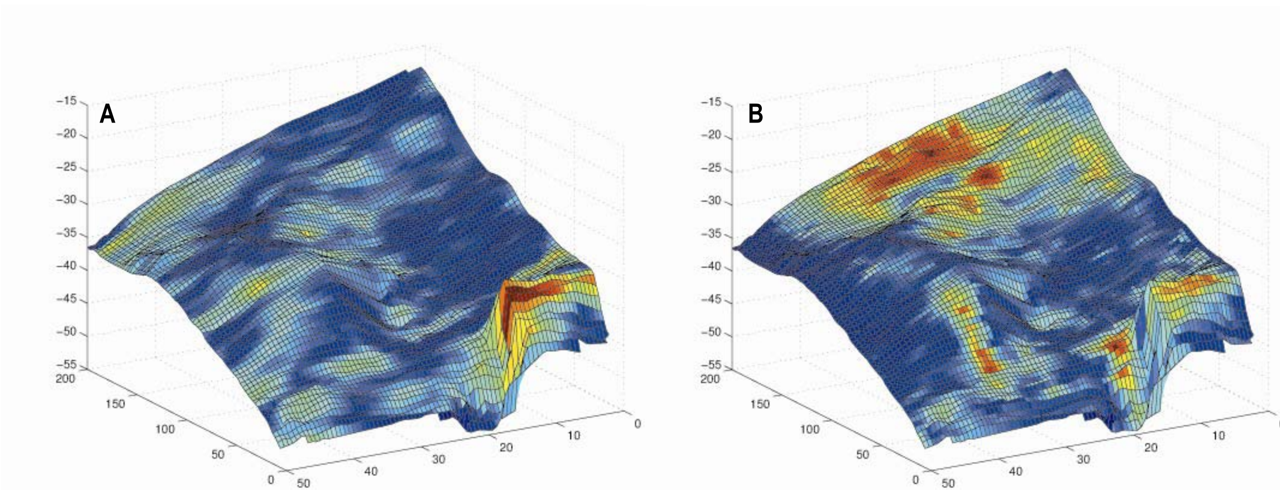

Figure 3. 1984-1999 difference projected onto the Top Ula horizon from ordinary difference cube (A) and computed using Curvelet domain adaptive subtraction (B)

Discussion

We presented an alternative method to compute difference cubes. Our method is not based on the actual subtraction of two datasets, risking the introduction of artefacts due to incoherent noise, possible misalignments, and differences in acquisition. Rather, it mutes events of one dataset with respect to the other based on coefficients in a sparse and local basis function decomposition. As a result, we do not introduce artefacts by subtracting misaligned events. Rather, events with a strong presence in the other dataset are muted. Even though the example shown is preliminary, it shows promise; as does the application of this method to adaptive subtraction of predicted multiples and migration, on which is reported elsewhere in these proceedings.

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References

[1] Candès, E.J. and Donoho, D.L., 2002, New tight frames of curvelets and optimal representations of objects with smooth singularities: technical report, Stanford, Department of Statistics.

[2] Candès, E.J. and Donoho, D.L.., 2000, Curvelets – a surprisingly effective nonadaptive representation for objects with edges: Curves and Surfaces. Vanderbilt University Press.

[3] Candès, E.J. and Donoho, D.L., 2000, Recovering edges in ill-posed problems: Optimality of curvelet frames: Annals of Statistics, **30**, 784–842.

[4] Candès, E.J. and Guo, F., 2002, New multiscale transforms, minimum total variation synthesis: Applications to edge-preserving image reconstruction: Signal Processing, 1519–1543.

[5] Coifman, R. R. and Donoho, D. L.., 1995, Translation-invariant de-noising: technical report, Stanford, Department of Statistics.

[6] Do, M. and Vetterli, M., 2002, Beyond wavelets: Academic Press.

[7] Donoho, D. L. and Johnstone, I. M., 1998, Minimax estimation via wavelet shrinkage: Annals of Statistics, **26**, 879–921.

[8] Herrmann, F.J., 2003, Multifractional splines: application to seismic imaging: Proceedings of the SPIE Technical Conference on Wavelets: Applications in Signal and Image Processing, 240–258.

[9] Jones, I.F., Folstad, P.G., and Kjos, E., 2002, Continuous high resolution velocity as a 4D attribute: Proceedings of the CSEG Conference.

[10] Mallat, S. G., 1997, A wavelet tour of signal processing: Academic Press.

[11] Vogel, C., 2002, Computational Methods for Inverse Problems: SIAM.