Practical aspects of nonstationary wavefield extrapolation

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Summary
The nonstationary phase-shift extrapolator provides a highly accurate wavefield extrapolator in laterally inhomogeneous media. However, because the operator depends on both the lateral spatial variable and its wavenumber, the transform between space and wavenumber cannot be implemented with the fast Fourier transform technique. Consequently, the computational effort is greater than that of stationary phase shift method. In this paper, a windowed nonstationary phase shift is implemented and practical issues such as the number of reference velocities are investigated. Numerical results show that computational efficiency is improved with little loss of accuracy.

Introduction
Implementation of wavefield extrapolation in the frequency-wavenumber domain has many advantages, e.g., the spatial derivatives are calculated accurately with the Fourier transform (Orsag, 1972) and ray-tracing is not necessary (Goran and Richards, 1991). Wavefield extrapolation is mostly based on the one-way wave equation, which comes from the factorization of Helmholtz equation. The exact factoring of the Helmoholtz equation for the apparently simple case of a homogeneous medium (constant velocity), results in a nonlocal one-way wave equation. The resulting phase-shift method (Gazdag, 1978) is an exact solution for the extrapolated one-way wavefield. The phase-shift method is attractive because the computation is very efficient due to its use of the fast Fourier transform (*FFT*) for all integrations. However, the requirement of constant velocity limits this method because seismic imaging must often cope with laterally inhomogeneities. Recently, Margrave and Ferguson (1999) presented a nonstationary phase-shift method for wavefield extrapolation based on pseudodifferential operator theory. With this operator, the Helmholtz equation can be approximately factorized into a one-way wave equation with high accuracy for the case of velocity varying laterally. The operator can be applied in either the frequency-wavenumber domain or the frequency-space domain. There are two elementary forms of nonstationary phase shift algorithms, i.e. the combination extrapolator (*PSPI* in the nonstationary limit) and the convolution extrapolator (nonstationary phase shift, *NSPS*). Unlike the stationary operator that is a function only of wavenumber, the nonstationary operator is the function of both variables. In this case, the *FFT* cannot be applied to the transform between the space and wavenumber domain so extra computational effort is required for nonstationary phase shift extrapolation. In order to gain computational efficiency, Margrave and Ferguson (1999) gave the windowed version of the nonstationary operator. However, the reduction of computational cost is still dependent upon the number of windows chosen, which in turn depends on the reference velocities chosen. For improved accuracy, the split-step-Fourier method (Stoffa, et al., 1990) can be applied within each window. The criteria of the threshold for allowed velocity variations within a window can be analyzed based on the error analysis of the split-step-Fourier method.

The method
Based on the nonstationary phase-shift, the approximate solution for wavefield extrapolation can be written as

$$\psi_{PSPI}(x,\Delta z,\omega)=\int\alpha(k_x,x,\Delta z)\varphi(k_x,z=0,\omega)\exp(ik_x x)dk_x \qquad (1)$$

and

$$\varphi_{NSPS}(k_x,\Delta z,\omega)=\int\alpha(k_x,x;\Delta z)\psi(x,z=0,\omega)\exp(-ik_x x)dx\,, \qquad (2)$$

for the *PSPI* and *NSPS* versions, respectively. Here, ψ and φ are wavefields representing velocity potential or pressure and its Fourier transform, and *x, z* and k_x are spatial coordinates and horizontal wavenumber, respectively. The nonstationary extrapolator, α*,* is defined as

$$\alpha(k_x,x,\Delta z)=\exp(\pm i\sqrt{\omega^2/v^2(x)-k_x^2})\Delta z \qquad (3)$$

The extrapolator α can be considered as a generalized phase-shift when the wavefield is propagating up and down along the z-axis (Wenzel, 1991). In the equations above, even though the integrals are very similar to the Fourier integral, they cannot be performed with the *FFT* because the extrapolator α is a function of both variables *x* and k_x. Apart from the calculation of the complex exponential elements, the direct integral of equation (1) or (2) involves N^2 operations (*N* is the number of spatial samples along the *x*-axis), which is much larger than cost of an *FFT* ($N\log_2 N$, for example). If *N* is very large, the computational burden will become prohibitive for practical use. Margrave and Ferguson (1999) suggested a windowed version of nostationary phase-shift, i.e. the distribution of horizontal velocity can be approximated by a piecewise constant velocity, v_j, and therefore, within each velocity window, an FFT can be applied. The windowed versions of *NSPS* and *PSPI* can be written as (Margrave and Ferguson, 1999)

$$\psi_{NSPS}(x,\Delta z,\omega)=IFT[\sum_j^K\alpha_{vj}(k_x,\omega)FT\{\Omega_j(x)\psi(x,0,\omega)\}] \qquad (4)$$

$$\psi_{PSPI}(x,\Delta z,\omega)=\sum_j^K\Omega_j(x)IFT\{\alpha_{vj}(k_x,\omega)FT\psi(x,0,\omega)\} \qquad (5)$$

where $\Omega_j(x)$ defines the velocity window and equals unity when $v(x)$ equals v_j and otherwise is zero, v_j is the reference velocity in j^{th} window and K is the number of windows. The cost of applying equations (4) or (5) is $Knlog_2N$ and therefore, we require $Klog_2N<N$, for computational efficiency. However, large deviations of the velocity from their reference value will affect the accuracy of the wavefield extrapolation. In order to overcome this dilemma, some approximate techniques can be applied, e.g. Born approximation (Huang et al., 1999) or split-step method (Hardin and Tappert, 1973), within each window. With the split-step method, equations (4) and (5) can be extended to

$$\psi_{NSPS}(x,\Delta z,\omega) = IFT[\sum_j \alpha_{vj}(k_x,\omega) FT\{\Omega_j(x) e^{B_j \Delta z} \psi(x,0,\omega)\}] \quad (7)$$

$$\psi_{PSPI}(x,\Delta z,\omega) = \sum_j^K \Omega_j(x) e^{B_j \Delta z} IFT[\{\alpha_{vj}(k_x,\omega) FT \psi(x,0,\omega)\}] \quad (8)$$

where $B_j = i\frac{\omega v_i}{2}(\frac{1}{v(x)^2} - \frac{1}{v_j^2})$ Comparing equations (7) and (8) with (4) and (5), the extra term applies first-order corrections for the errors coming from velocity deviations, which allows larger velocity deviations than equation (4) and (5). The threshold for the velocity deviations from their reference velocity can be analyzed with the relative phase error between the exact phase $\sqrt{\frac{\omega^2}{v(x)^2} - k_x^2}$ and the applied phase $\sqrt{\frac{\omega^2}{v_j^2} - k_x^2} + \frac{\omega}{2v(x)}(\frac{1}{v(x)} - \frac{v(x)}{v_j^2})$. Define $k=\omega/v(x)$, $sin\theta=k_x/k$ and the relative velocity deviation is $\delta\sigma= (v(x)-v_j)/v_j$, this relative phase error, ε_r can be written as

$$\varepsilon_r = \{cos\theta - \sqrt{(1+\delta\sigma)^2 - sin^2\theta} + \delta\sigma\}/cos\theta \quad (9)$$

Equation (9) can be used to calculate the relative phase error as a function of the propagation angle for given relative velocity perturbations. Figure 1 shows relative error for relative velocity perturbations from *5%* to *30%*. The errors increase with increasing angles. If the relative velocity perturbations are less than *10%*, then for propagation angles less than *60°*, the relative errors are below *5%*.

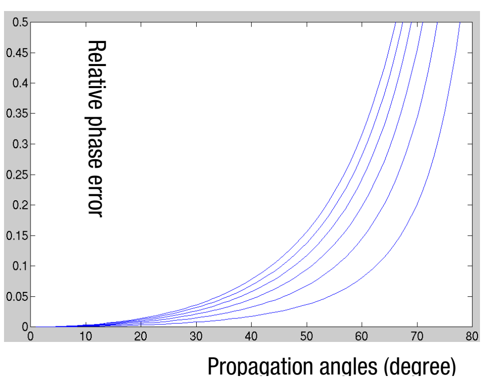

Figure 1. The relative phase errors vs propagation angle. Relative velocity is varied from 5% to 30% with increment of 5%.

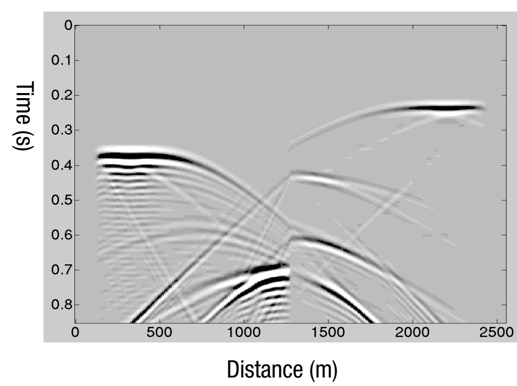

Figure 2. Synthetic data by the fourth-order, staggered-grid, finite-difference method.

Numerical examples

The first model is shown in Figure 3 for the test of post stack migration application. The exploding reflector shown in Figure 3a is used for modeling the wave propagation in the velocity model shown in Figure 3b. The synthetic data (Figure 2) was generated by the fourth-order, staggered-grid, finite-difference method. In Figure 2, the data in the left part contains coherent noise (head waves off the vertical interface) because the velocity there is much lower that that of the right part. There is also more numerical dispersion on the left. However, the existence of these noises can also be used to test the flexibility of the method.

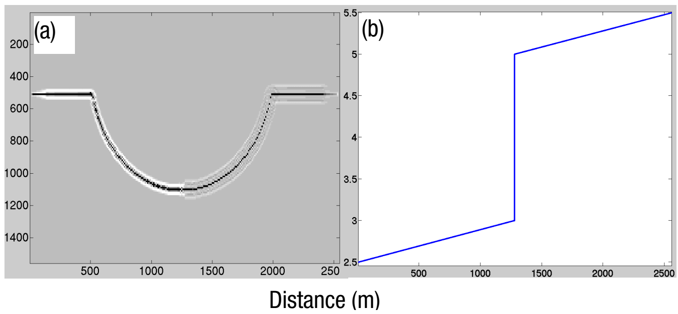

Figure 3. (a) The exploding reflector and (b) the velocity model.

The result shown in Figure 3a is produced the windowed nonstationary phase-shift with two windows that cover the left and right parts, respectively. The migration used a 20m depth step. Comparing with the more accurate result (Figure 3b) produced by the eigenfunction decomposition method (Yao and Margrave, 1999) shows that the simplified windowed nonstationary extrapolator works very well.

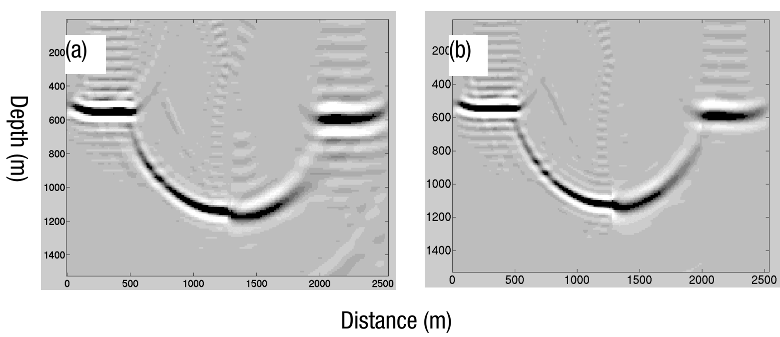

Figure 4. Results from (a) windowed method and (b) eigenfunction decomposition method.

The next test is on the data from the Marmousi model (Figure 5 (a)) for prestack migration (Bourgeois et al., 1991). The windows are generated by the criteria that velocity variations must be less than 10% relatively. The result is shown in Figure 5(b).

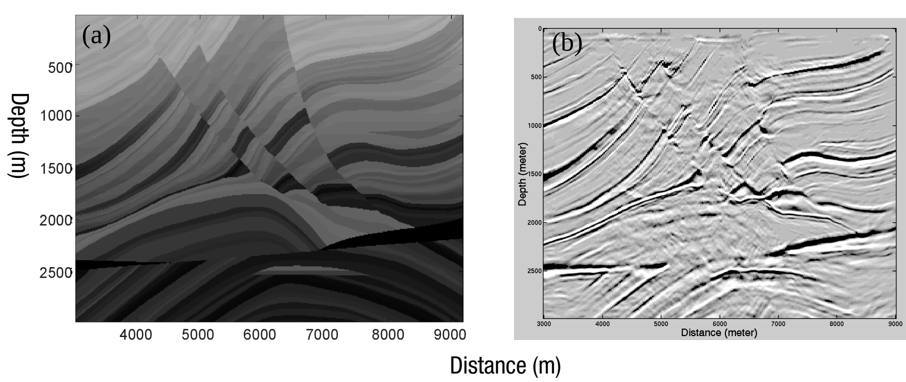

Figure 5. Marmousi model (a) and migration result image (b).

Comparing the migrated result with the original velocity mode shows that all of the major seismic markers are present in the migrated image.

Conclusions

The nonstationary phase-shift method can handle wavefield extrapolation in strong laterally variant velocity media. The windowed version makes the practical application possible. It speeds up the original nonstationary phase-shift but retains accuracy. Numerical results show that when the velocity variations are less than 10% in each window, the method works well.

Acknowledgment

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