Separable offset least-squares DSR migration of incomplete data

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Summary

Offset separable phase-shift DSR ("Double-Square-Root") migration is used to simultaneously compute sets of constant-offset sections. The offset separable migration is implemented as a non-recursive algorithm and is valid only for laterally invariant media (or time migration). Noise and missing or irregularly sampled reflection data reduce the quality of the common midpoint (CMP) gathers significantly. Least-squares (LS) migration with a smoothing constraint in the model domain along the offset dimension improves the signal to noise ratio and restores the continuity of the CMP gathers.

In separable offset LS migration the forward (modeling) and the adjoint (migration) operators are applied iteratively to minimize an objective function that penalizes discontinuities along offset that can be attributed to noise and missing data.

The modeling/migration operators are very efficient due to the simultaneous computation of the constant offset sections. The LS algorithm is easy to implement in a parallel computer environment.

Significant improvements of the LS migrated images occur after only a few iterations which makes the proposed technique feasible for processing large data sets.

Introduction

Least-squares (LS) migration based on Kirchhoff modeling/migration operators has been proposed in the literature to account for uneven subsurface illumination and to reduce imaging artifacts due to irregularly and/or coarsely sampled seismic wavefields (Nemeth et al., 1999; Duquet et al., 2000). Duquet et al. (2000) demonstrate how to further improve the artifact reduction of LS migration by applying a smoothing constraint along the offset domain. Kuehl and Sacchi (2001a) show that the concept of least-squares migration for incomplete data can also applied to phase-shift pre-stack "Double-Square-Root" (DSR) migration by introducing a data weighting operator to account for the missing data. In order to apply the smoothing constraint in DSR migration the migrated constant-offset sections need to be kept separate. Therefore DSR migration is implemented as a non-recursive algorithm (Popovici, 1995) which precludes the use of correction techniques that attempt to accommodate lateral velocity variations (e. g. Phase-Shift-Plus-Interpolation (Gazdag and Squazzero, 1984); Split-step migration (Stoffa et al., 1990); Non-Stationary –Phase-Shift migration (Margrave and Ferguson, 1984)). In a companion paper (Kuehl and Sacchi, 2001b) we demonstrate how one can apply a similar smoothing constraint like the one described to phase-shift depth migration by employing a constant angle imaging (CAI) condition (Mosher and Foster, 2000).

Theory

Inverse scattering theory provides an instructive framework for deriving phase-shift DSR modeling and migration operators in midpoint-offset coordinates (Stolt and Benson, 1986; Clayton and Stolt, 1981).

The adjoint modeling/migration operator pair is used to invert the following linear system in a least-squares sense:

$$\mathbf{d} = \mathbf{A}\mathbf{m} + \mathbf{n},$$

where $\mathbf{d}$ is the (noisy) incomplete data, $\mathbf{A}$ the separable-offset DSR modeling operator in midpoint-offset coordinates and $\mathbf{m}$ the model that contains a set of constant-offset image gathers $\mathbf{m}_{ih}$. The error term $\mathbf{n}$ represents modeling errors, missing data and noise (see also Duijndam, 1988).

We minimize the following objective function using a conjugate gradient (CG) algorithm:

$$\min F(\mathbf{m}) = \| \mathbf{W}(\mathbf{d} - \mathbf{A}\mathbf{m}) \|_2 + \lambda \sum_{ih} \| \mathbf{m}_{ih} - \mathbf{m}_{ih-1} \|_2,$$

where $\mathbf{W}$ is a diagonal weighting operator with zero weights for dead data traces and non-zero weights for live traces according to their noise level (Kuehl and Sacchi, 2001a). The second term with the trade-off parameter λ imposes a relative smoothing constraint that suppresses undesired discontinuities in the offset direction.

Example

We have generated a midpoint-offset data set using Kirchhoff modeling based on a simple syncline structure. We note that to test the robustness of the inversion algorithm the synthetic data has been generated by a different type of operator than the one used in the inversion algorithm. The Figures 1 and 2 show the migrated midpoint gathers and a close-up of 6 midpoints of the complete noisy data, respectively. In Figures 3 and 4 the effect of missing data (60% of the data has been randomly removed) on the image quality is demonstrated. The midpoint gathers are discontinuous along offset and spurious events are apparent. The least-squares separable offset migration has restored continuity along offset and reduced the noise level and much of the artifacts due to missing data after 4 iterations of the CG algorithm (Figures 5 and 6). Since only two adjacent offsets are considered in the smoothing term of the objective function the offset move-out due to an incorrect migration velocity is preserved in LS migration (Figures 7 and 8).

Conclusions

Separable offset least-squares DSR migration with relative smoothing constraints along the offset domain reduces artifacts due to missing data and enhances the signal to noise ratio of CMP gathers. The separable-offset DSR modeling/migration operators are very efficient in computing time but restricted to media with laterally invariant background velocities or time migration.

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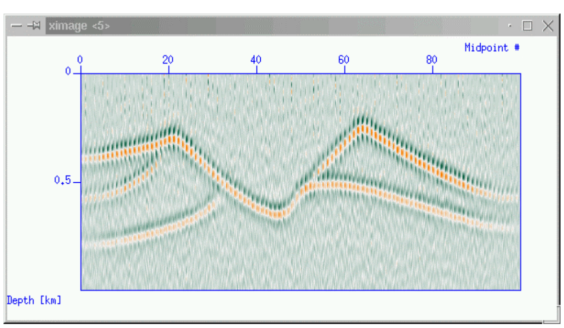

Figure 1: Midpoint gathers of 32 constant offset sections that have been simultaneously computed by the separable offset DSR Migration

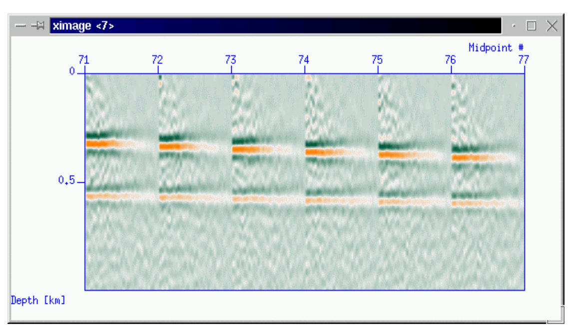

Figure 2: Midpoint gathers 71 to 77 of the migrated noisy data (S/N = 10).

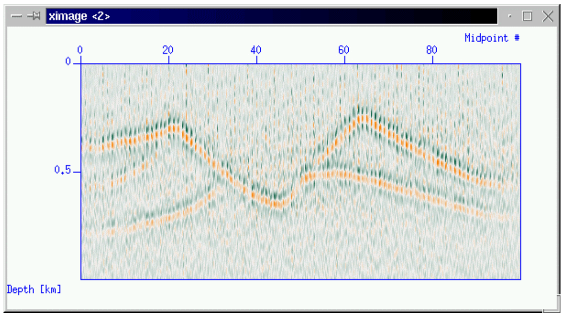

Figure 3: Midpoint gathers of the reduced and noisy data (60% of the prestack data traces have been randomly set to zero).

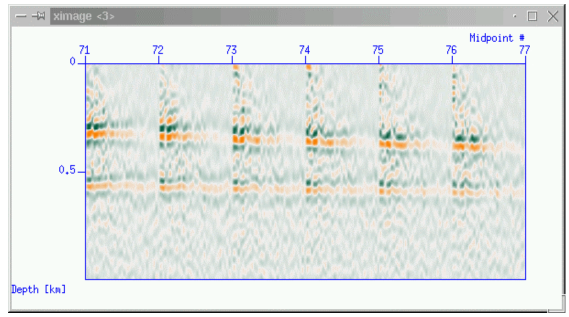

Figure 4: Midpoints gathers 71 to 77 of the migrated incomplete and noisy data set.

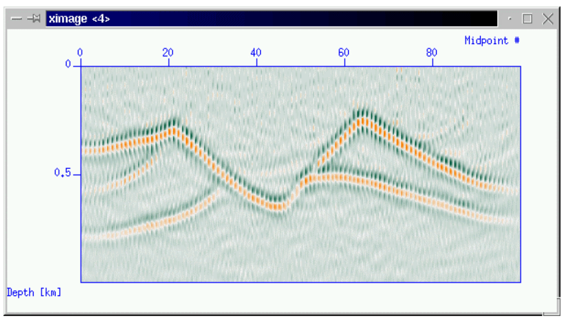

Figure 5: All midpoint gathers after 4 CG iterations of the incomplete noisy data set.

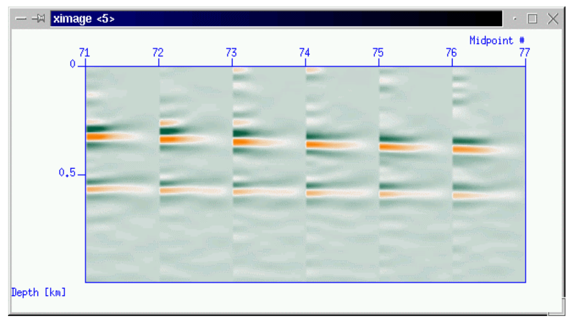

Figure 6: Midpoint gathers 71 to 77 after 4 CG iterations of the incomplete noisy data set

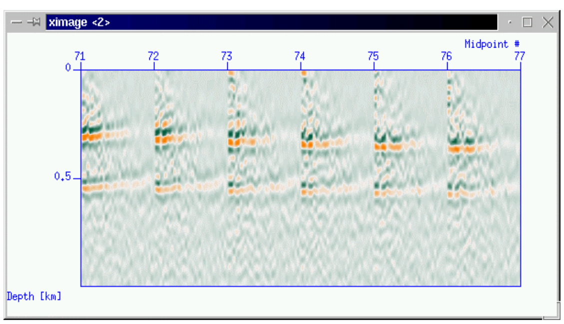

Figure 7: Migration of the constant offsets with a migration velocity that is 10% too low.

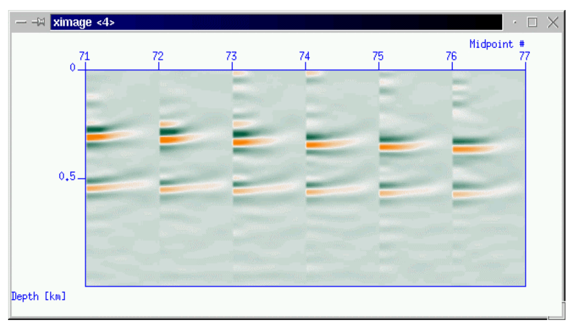

Figure 8: Least-squares migration after 4 iterations with incorrect velocities. The move-out due to the low migration velocity is preserved.