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ANGLE OF INCIDENCE AS A FUNCTION OF SOURCE-RECEIVER OFFSET OVER A DIPPING REFLECTOR; AN EXACT EXPRESSION FOR VSP APPLICATIONS

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SHORT NOTE

INTRODUCTION

Reflection amplitudes are intimately connected to the angle of incidence. In seismology, however, the angle of incidence is often difficult to establish. Partially, because of this difficulty it is more common to consider Amplitude Variations as a function of a lateral source-receiver Offset (AVO) rather than Amplitude Variations as a function of the Angle of incidence (AVA). Computational modelling and theoretical analysis, nevertheless, require the knowledge of angles of incidence in order to relate them directly to various forms of Zoeppritz equations (e.g., Aki and Richards, 1980). Furthermore, although a lateral source-receiver offset is easily established based on field acquisition parameters, the angle of incidence requires a more involved calculation.

This Short Note provides explicit and exact expressions which can be used in AVA studies using the Vertical Seismic Profile (VSP). The expressions can be conveniently used in planning an AVA/AVO survey while designing source-receiver configurations for a given range of angles of incidence. The approach considers single-mode waves reflecting from a dipping planar reflector in an isotropic and homogeneous medium.

METHOD AND RESULTS

Horizontal reflector

Consider a trajectory of a signal between a source at the surface and the horizontal and planar reflector at a depth, H, (Figure 1). The angle of incidence, θ, can be expressed by:

$$\tan\theta = \frac{r}{H}, \tag{1}$$

where r is the lateral distance between the source and reflection point. Similarly for the angle of reflection, θ, one can write:

$$\tan\theta = \frac{X-r}{H-Z}, \tag{2}$$

where X is the lateral source-receiver offset and Z is the depth of the receiver. Note that single-mode propagation in isotropic media is assumed, hence angles of incidence and reflection are equal. Solving equations (1) and (2) for the common variable, r, and equating them yields:

$$H\tan\theta = X-(H-Z)\tan\theta\ . \tag{3}$$

Thus the angle of incidence or reflection is:

$$\theta = \arctan\frac{X}{2H-Z}\ . \tag{4}$$

Note that equation (4) could be obtained immediately by using a mirror image, i.e., by placing a receiver the same distance below the reflector and connecting the source and the receiver by a straight line. The presented approach, however, provides a background for the case of dipping reflector.

Dipping reflector

Consider a VSP survey recording seismic reflections from a planar but dipping reflector (Figure 2). As before, let the lateral offset between the source and receiver be X, the vertical depth of the receiver in the wellbore be Z and the reflector be vertically below the receiver at the depth H. Assume the dip of the reflector to be known, e.g., from surface-seismic or dip-meter information, to be α.

To render the approach similar to the case considered for the horizontal layer, one can redraw the image (Figure 3). In the new image, marked with dotted lines and described with parameters denoted by small letters, one obtains (see Appendix A), without any loss of generality, a picture and expressions similar to the case of the horizontal reflector.

Thus:

$$x = \frac{\cos(\theta-\alpha)}{\cos\theta}(X+Z\tan\alpha), \tag{5}$$

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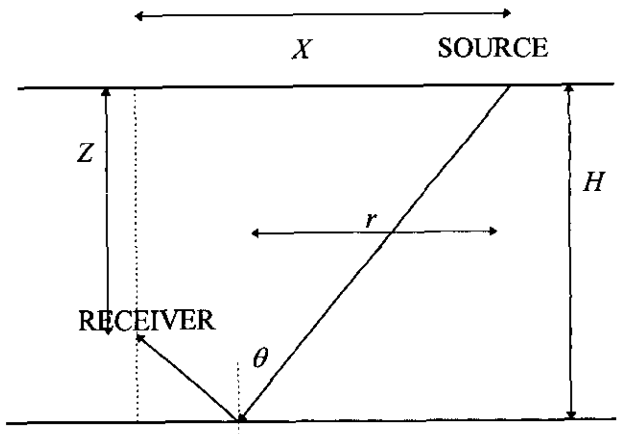

Fig. 1. Horizontal-layer case. The symbol r denotes the lateral distance between the source and the reflection point.

$$z = \frac{Z}{\cos\alpha},\tag{6}$$

and

$$h = \frac{Z}{\cos\alpha} + (H - Z)\cos\alpha.\tag{7}$$

One can notice that if $\alpha = 0$, i.e., the reflector is horizontal, then $x = X$, $z = Z$ and $h = H$, as expected. Considering the angle of incidence one can write:

$$\tan\theta = \frac{r}{h} = \frac{r}{\frac{Z}{\cos\alpha} + (H - Z)\cos\alpha}.\tag{8}$$

Considering the angle of reflection one can write:

$$\tan\theta = \frac{x - r}{h - z} = \frac{\frac{\cos(\theta - \alpha)}{\cos\theta}(X + Z\tan\alpha) - r}{(H - Z)\cos\alpha}\tag{9}$$

From this the explicit solution for the incidence or reflection angle, θ, (see Appendix B) is

$$\theta = \arctan\frac{X\cos\alpha + Z\sin\alpha}{(2H - Z)\cos\alpha - X\sin\alpha}.\tag{10}$$

One can notice that if the reflector is horizontal, i.e., $\alpha = 0$, then, as expected, equation (10) is reduced to equation (4).

Equation (10) can be used for sources located both updip and downdip from the receiver (see Appendix C). The dip angle in Figures 2 and 3 is assumed to be positive, i.e., measured from the horizontal axis in the counter-clockwise direction.

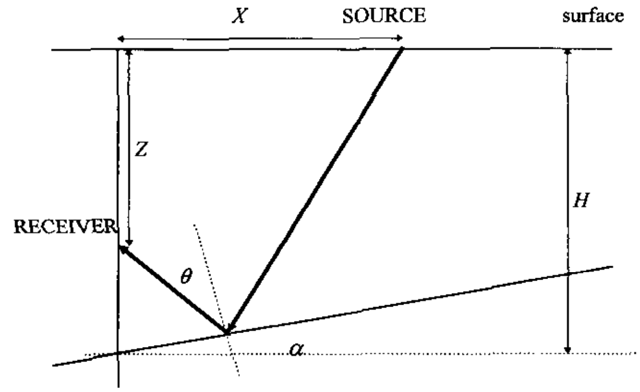

Fig. 2. VSP data acquisition over a dipping reflector. Source on the surface is located updip of the receiver.

Example

Consider a vertical wellbore which penetrates a dipping reflector at H = 1000 metres. Assume that a receiver is placed at Z = 900 metres, i.e., 100 metres above the reflector. Figure 4 shows angle of incidence, θ, as a function of dip angle, α, and lateral source-receiver offset, X. (Note that using expression (10) one can also investigate the variation of incidence angle with the receiver depth, Z.)

Discussion and Conclusions

Investigating variation of reflection amplitude with the angle of incidence using VSP data is generally performed with the receiver positioned close to the reflector and examining different angles of incidence by varying the location of the source (e.g., Coulombe et al., 1996). For rigorous investigations using Zoeppritz equations it is important to know the angles of incidence corresponding to a given source-receiver pattern. Equation (10) can be conveniently used to provide a tool for such an analysis. Moreover, knowing the value of incidence angle, θ, from equation (10) one can calculate the value of r from equation (8) or (9). Hence, one knows the

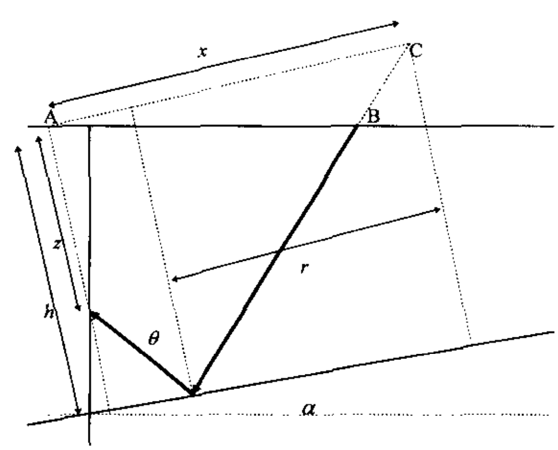

Fig. 3. Modified image of a dipping reflector, which renders the geometrical approach more akin to the horizontal-layer case.

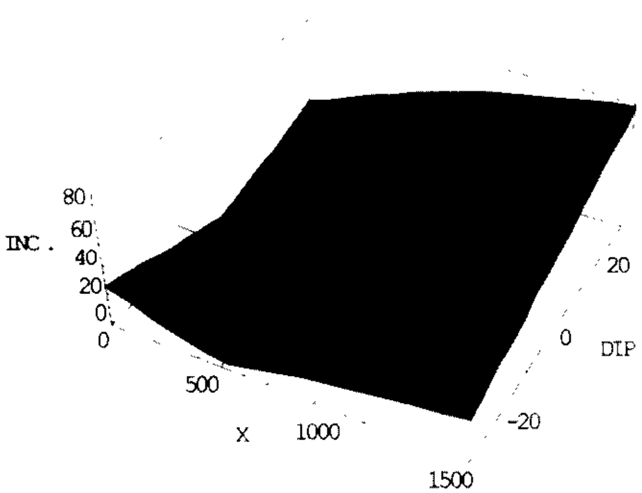

Fig. 4. Angle of incidence/reflection, θ, as a function of lateral source-receiver offset, X, (0m to 1500m) and the reflector dip, (-30° to 30°) with the pivoting point directly below the wellbore. The vertical wellbore penetrates the reflector at H = 1000 metres while the receiver is located at a depth of Z = 900 metres.

location of the reflection point – important information in establishing local lithological characteristics based on variation of reflection amplitudes.

This paper presents exact and explicit formula to calculate the angle of incidence on a dipping interface for a ray travelling between source and receiver located at different elevations. The formula may be useful in designing AVO/AVA studies for VSP acquisition and in analyzing the results in the scope of Zoeppritz equations.

More interesting and useful work can be done to include mode conversions and anisotropy. In some ways both approaches could be similar since both cases imply that angles of incidence and reflection do not have to be equal (e.g., Tessmer and Behle, 1988). In the case of wave propagation in anisotropic media one might have to consider concepts of both phase and group velocities thus involving a more complicated approach. Nevertheless, since the variable r remains common to both incidence and reflection angles, even for mode conversions in anisotropic media, a development of more general expressions seems feasible.

Other elements can be included by considering multilayer isotropic and three-dimensional cases, i.e., the saggital plane not paralleling the dip direction. In such cases, however, exact and explicit expressions might be impossible to obtain and one might have to resort to numerical methods.

An approach which would bring mathematical expressions closer to geophysical reality has been suggested by B. Goodway (pers. comm., 1997). Instead of using the single layer with constant velocity one could attempt to derive an exact and explicit equation for a case of velocity, v, varying linearly with depth, z, i.e.,

$$v(z) = v_0 + kz, \qquad (11)$$

where v_0 is the value of velocity at the surface, and k is a constant. It appears that in many areas of Alberta one can establish values of v_0 and k which agree well with observed data.

The suggested approach, which we are presently investigating (Epstein and Slawinski, 1998), bears an interesting resemblance to the brachistochrone problem proposed and solved by Jean Bernoulli in 1696 (e.g., reprinted in Smith, 1959). Notably, the solution of this problem led to the calculus of variations. As a historic anecdote, Bernoulli's proposal of the problem was preceded by an interesting statement, namely, that "the question proposed does not, as it might appear, consist of mere speculation having therefore no use".

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Appendix A

Source located updip

In Figure 3 consider the triangle ABC with the hypotenuse denoted by x. The obtuse angle is $\pi/2+\theta-\alpha$. One acute angle is α, while another is $\pi/2-\theta$. Using the Sine Law one may write:

$$\frac{x}{\sin\left(\frac{\pi}{2}+\theta-\alpha\right)} = \frac{X+Z\tan\alpha}{\sin\left(\frac{\pi}{2}-\theta\right)}. \qquad \text{(A-1)}$$

Simplifying by using trigonometric reduction formulæ one obtains:

$$\frac{x}{\cos(\theta-\alpha)} = \frac{X+Z\tan\alpha}{\cos(\theta)}, \qquad \text{(A-2)}$$

and solving for x yields equation (5), i.e.,

$$x = \frac{\cos(\theta - \alpha)}{\cos\theta}(X + Z\tan\alpha)\cdot \tag{A-3}$$

Appendix B

Explicit-form solution

Solving both equations (8) and (9) for r, equating them and rearranging to place θ-dependent terms on the left-hand side yields:

$$\sec(\theta - \alpha)\sin(\theta) = = \frac{(X + Z\tan\alpha)}{\frac{Z}{\cos\alpha} + 2(H - Z)\cos\alpha}. \tag{B-1}$$

The angle of incidence/reflection can be calculated explicitly as shown in equation (10), namely:

$$\theta = \arctan\frac{X\cos\alpha + Z\sin\alpha}{(2H - Z)\cos\alpha - X\sin\alpha}. \tag{B-2}$$

Appendix C

Source located downdip from receiver

For a source location downdip of the receiver one can consider a figure analogous to Figure 3. In such a case equations (6) and (7) are the same but the equivalent triangle ABC, used for the expression for x by the Sine Law, is located immediately below the surface and has different angles. Thus one obtains:

$$x = \frac{\cos(\theta - \alpha)}{\cos\theta}(X - Z\tan\alpha). \tag{C-1}$$

Notice that if the value of α is set to a negative number for the source located downdip from the receiver, the equations (5) and (C-1) are identical. Also, note that due to symmetric trigonometric functions equations (6) and (7) are independent of the sign of α.