CANADIAN JOURNAL OF EXPLORATION GEOPHYSICS
VOL. 32, NO. 2 (DECEMBER 1996), P. 90-103

A COUPLED METHOD OF MIGRATION AND TOMOGRAPHY TO ESTIMATE SUBSURFACE VELOCITY STRUCTURE

EIICHI ASAKAWA[1]

ABSTRACT

A new method to estimate both subsurface structure and velocity distribution using seismic traveltimes is proposed. Reflection tomography is often used to estimate subsurface velocity distribution and it requires an initial estimate of the positions of reflectors because raytracing has to be performed before tomographic inversion. Most conventional reflection tomography determines the position of reflectors using information such as well log data and seismic time sections. In this paper, a kind of kinematic migration of the traveltime data is proposed in order to estimate the reflector positions. Since only traveltime data picked from the prestack seismic traces are used in this procedure, the method proposed in this paper is very efficient. This approach eliminates the need for time consuming migration and the difficulty of raytracing of the reflected wave in the cell model used in tomographic inversion.

The basic concept is described and then applied to a numerical experiment data with the detailed discussion. Finally this method is applied to the data which are obtained with a scaled model.

INTRODUCTION

Prestack depth migration is widely used to image complex structure. Accurate estimation of velocity structure is required before applying prestack depth migration. The conventional way to estimate velocity structure is to use stacking velocities. However, CMP velocity analysis does not work well in media which have lateral velocity variations.

Recently, reflection tomography has been applied to the estimation of the velocity structure for prestack depth migration. The tomographic approach has the advantage of handling the global velocity variations in cell models and it is suitable for cases where the lateral velocity variation is severe.

In applying reflection tomography, the positions of the reflectors must be estimated before inversion of velocity distribution because it is essential to find the reflected raypaths. However, most of the conventional methods handle velocity variations by estimating the position of the reflector using well logs, check shots or interpreted horizons.

Bishop et al. (1985) first proposed reflection tomography to estimate both the velocity and the reflector position simultaneously. Stork and Clayton (1991) proposed coupled inversion of velocity and reflector position. Their method combines transmission tomography and wave equation migration. It requires considerable computer time in the migration portion.

A new reflection tomography approach is proposed in this paper. Only the two-way traveltimes of the reflection events picked from the prestack seismic data are analyzed to estimate both the reflector and velocity distribution. This method consists of two processes. The first process is a kinematic migration to estimate the positions of reflectors. Because it does not use full migration but only a kinematic migration of traveltime data, it is a very efficient procedure. The migration used in this paper is similar to Kirchhoff migration, and it is essential to estimate the traveltimes on grid points in the depth section. Asakawa and Kawanaka (1993) proposed an efficient and accurate raytracing method in a cell model called Linear Traveltime Interpolation (LTI), which is suitable for tomographic analysis. LTI is applied to calculate traveltimes on grid points in this process. The position of each reflector is estimated statistically from the depth section on which the migrated points of the observed traveltimes are plotted.

The second process is reflection tomography to estimate velocity variations. It is necessary but difficult to calculate the raypath reflected at a certain reflector in a cell model. The shooting method does not work well in cases of large velocity variations. In this paper, I present algorithm new raytracing for reflected waves at a certain reflector. Matsuoka and Ezaka (1992) proposed the raytracing of reflected waves using the reciprocity principle. Their method is an efficient way to estimate the reflection point on a reflector using traveltimes at grid points. LTI and raytracing by reciprocity are coupled to estimate the reflected raypaths

Manuscript received by the Editor October 20, 1995; revised manuscript received April 25, 1996; accepted May 28, 1996.
[1]Japan National Oil Corporation, 1-2-2 Hamada, Mihama-ku, Chiba, JAPAN 261
Present address: Japex Geoscience Institute, Inc., 2-2-20, Higashi-shinagwa, Shinagwa-ku, Tokyo 140 JAPAN
This work has been carried out in the cooperative research project between JNOC and JAPEX. The author thanks Tadeusz Ulrych of University of British Columbia and Larry Lines of Memorial University for their suggestions and critical reading of the manuscript.

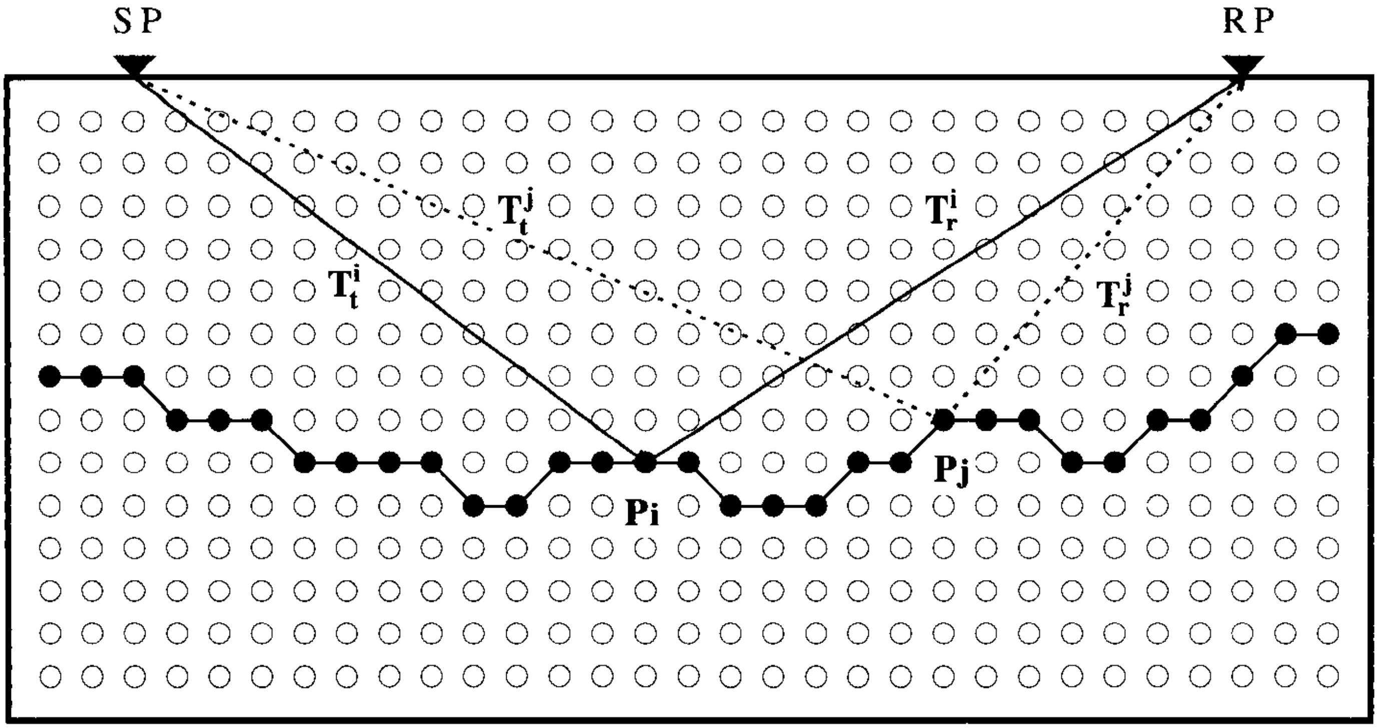

Fig. 1. The concept of equi-two way time curve. Pi is reflection point. Tt is transmission traveltime from shot point to reflection point. Tr is reflection traveltime from reflection point to receiver point. The equi-two way time curve consists of the points which satisfy the condition that the observed traveltime is equal to Tt+Tr.

in a tomographic geometry. It overcomes the trial and error approach and it is efficient.

Basic Concept

The method proposed in this paper consists of two processes. The first estimates the position of a reflector using traveltime data. The second is the estimation of the velocity distribution.

The first process requires the conversion of the observed traveltimes to depth. The basic idea is the same as used in kinematic Kirchhoff migration. The points which give the observed reflection traveltime for a certain pair of shot and receiver points lead to a curve in the depth section. This curve is called a equi-two way traveltime curve in this paper. The points on the curve represent all possible reflectors for a given traveltime. In reflection seismics, a number of shot and receiver pairs give such curves on the depth section. The most probable reflector could be extracted statistically.

It is essential to estimate the equi-two way traveltime curves which correspond to the observed traveltimes. The reflection event comes from the points where the summation of traveltimes from the shot to the point and from the receiver to the point is equal to the observed traveltime, as shown in Figure 1. The points that satisfy the condition form a curve in the depth section. In a simple case where the velocity is constant, the curve becomes an ellipse with the foci at the shot and receiver points. However, in general, velocity variation exists and the following approach is proposed. Assume that the shot point is SP and receiver point is RP and that the observed traveltime is T. At first, the traveltimes from SP are calculated at all grid points in the depth section. This traveltime distribution is called the transmission traveltime map. Next, traveltimes from RP are calculated at all grid points leading to what we call the reflection traveltime map. The two-way traveltime map is obtained by

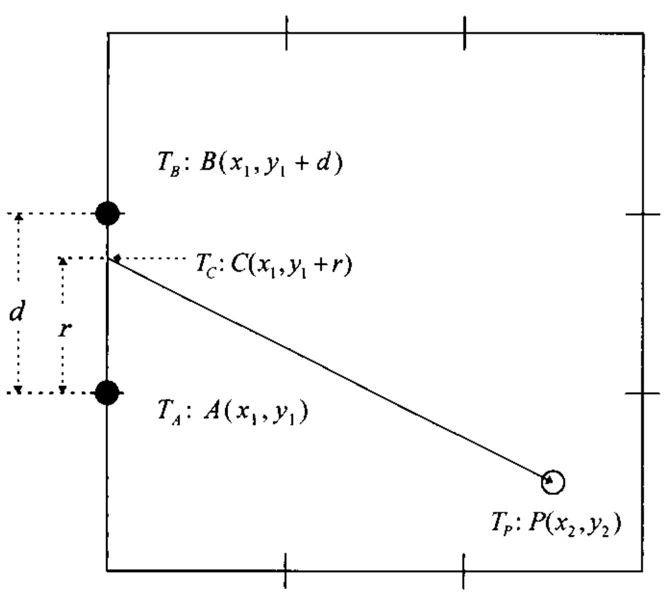

Fig. 2. A raypath crosses the line segment AB at point C and arrives at the point P inside the cell. Point C is located at distance r from point A. The traveltime at point C is assumed to vary from T_A to T_B linearly as r changes from 0 (at point A) to d (at point B).

adding the transmission and reflection traveltime maps. The equi-two way traveltime curve is estimated by picking the grid points which have the two way traveltime equal to T in the two way traveltime map.

A number of equi-two way traveltime curves which correspond to the shot-receiver pairs can be plotted on the same depth section and the most probable reflector is estimated from these curves which are expected to be numerous near the true reflector.

The second process is to estimate the velocity distribution by means of reflection tomography. Raytracing is performed using the reflector position estimated by the first process, and then tomographic inversion is performed to minimize the difference between the calculated traveltimes and the observed traveltimes. The velocity distribution is updated in this process.

In reflection tomography it is essential to compute the reflected raypaths from the assumed reflector. In general, it is difficult to compute a reflected raypath in a cell model. Matsuoka and Ezaka (1992) proposed a theory to estimate the reflection point along a certain reflector on the basis of reciprocity and Fermat's Principle. Based on their approach, the reflection point can be obtained by finding the minimum two-way traveltime along the estimated reflector. Combining the traveltime computation by LTI and their theory, the reflection point on the reflector can be obtained. The raypath from shot point to the estimated reflection point is calculated by applying LTI backward process in the transmission traveltime map and the raypath from the reflected point to the receiver point is calculated using the reflection traveltime map. The two-way reflected raypath is obtained by connecting the two raypaths.

Linear Traveltime Interpolation

In this paper, Linear Traveltime Interpolation (LTI) is used to calculate traveltimes and raypaths in a depth section. The LTI is formulated for the cell model based on Fermat's principle. The traveltime or raypath is calculated only on the cell boundaries and a raypath is considered to be straight in a certain cell with uniform velocity. This approach is suitable for tomographic analysis. The LTI consists of a forward and backward process. In the forward process, the calculation of traveltimes on the cell boundaries from a shot point is performed. In the backward process, the raypath from a certain receiver point to the shot point is traced.

Once the forward process has been performed, the traveltime at an arbitrary point in the depth section can be calculated by using the traveltimes on the boundaries of the cell which contains the calculated point. Consider a raypath crossing segment AB on a certain cell boundary and reaching the point P inside the cell as shown in Figure 2. The equation to calculate the traveltime on the point P is derived by modifying LTI's basic equation for the forward process. Assume that traveltimes at points A, B, C and P are T_A, T_B, T_C and T_P, then T_P is given by the following equation. (The point C is the crossing point.)

$$T_P = T_C + S\sqrt{(x_2 - x_1)^2 + (y_2 - y_1 - r)^2} \qquad (1)$$

where S is the slowness inside the cell.

Assuming that the traveltimes vary linearly on the cell boundary AB, so that the traveltime at point C can be calculated by linear interpolation of the traveltimes at points A and B. Hence, with C at a distance r from A, T_C will vary linearly from T_A to T_B linearly as r varies from 0 to d according to the equation,

$$T_C = T_A \frac{d-r}{d} + T_B \frac{r}{d}. \qquad (2)$$

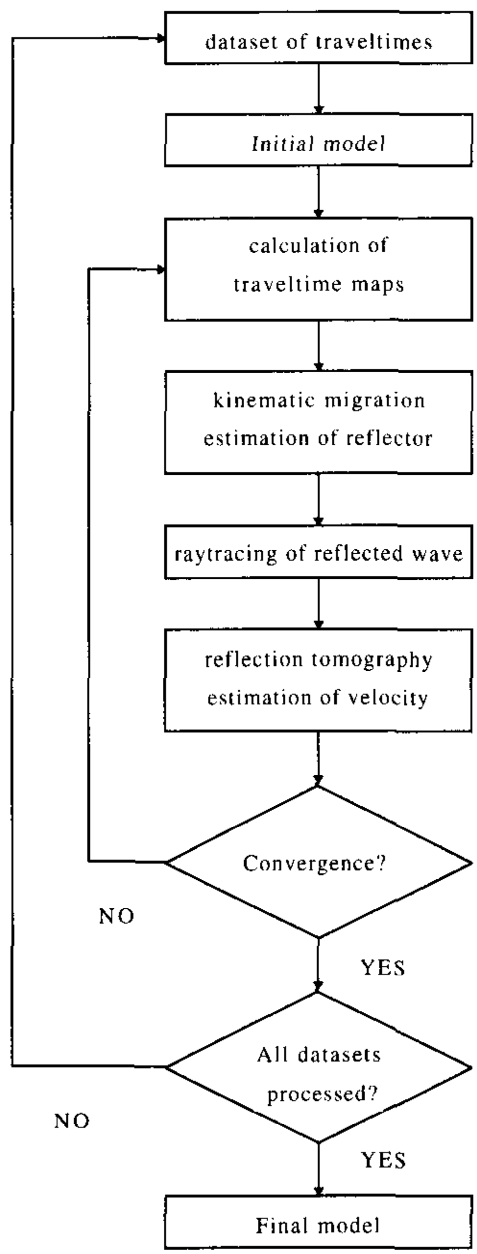

Fig. 3. Processing flowchart of the proposed method.

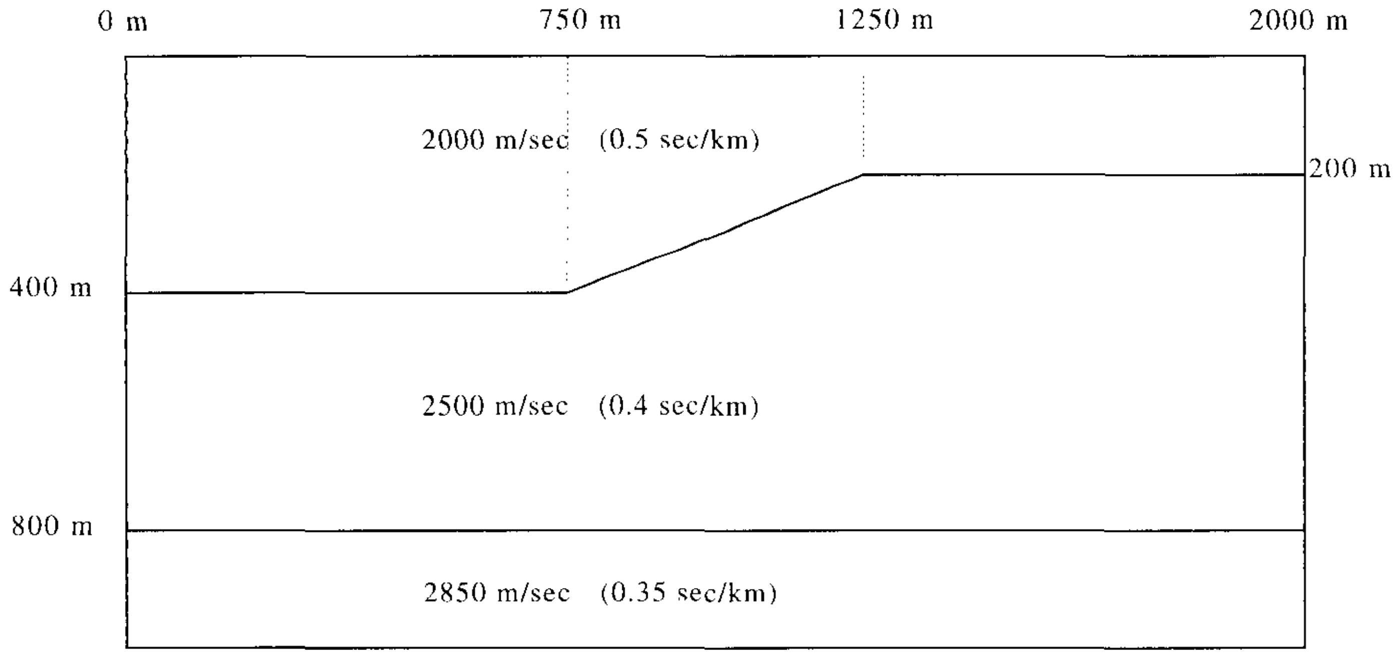

Fig. 4. Velocity structure of the simulation model. It consists of three layers. Each layer has constant velocity (slowness).

Hence, (1) can be rewritten as

$$T_P = T_A \frac{d-r}{d} + T_B \frac{r}{d} + S\sqrt{(x_2 - x_1)^2 + (y_2 - y_1 - r)^2}. \quad (3)$$

Since Fermat's principle states that the minimum traveltime at point P is the true traveltime, the true traveltime at point P and crossing point C can be obtained by differentiating (3) with respect to r.

$$T_P = T_A + \Delta T \cdot \frac{y_2 - y_1}{d} + \frac{x_2 - x_1}{d} \cdot \sqrt{S^2 \cdot d^2 - \Delta T^2} \quad (4)$$

$$r = \frac{\Delta T}{\sqrt{S^2 \cdot d^2 - \Delta T^2}} - (y_2 - y_1) \quad (5)$$

where $\Delta T = T_A - T_B$. Equation (4) is applied to all the segments on the cell boundaries and many traveltimes at P crossing

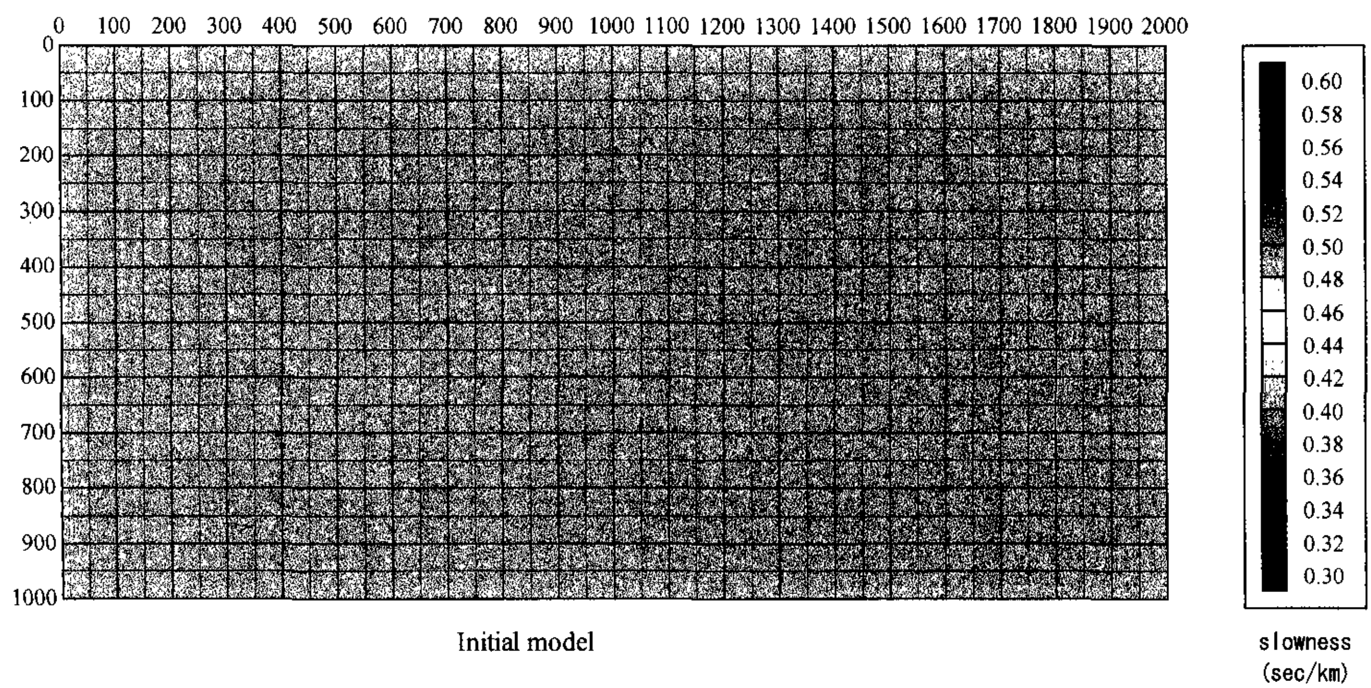

Fig. 5. Initial model.

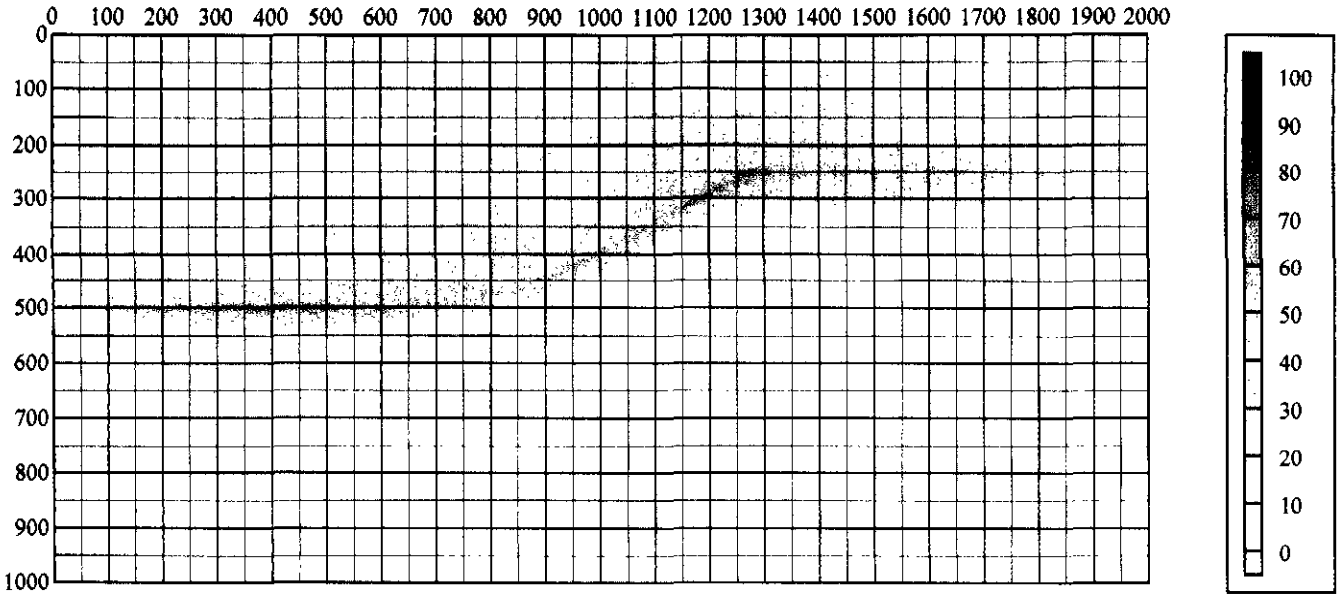

Fig. 6. The distribution of possible reflection points. This figure shows the histogram of the possible reflection points on each gridded point in the depth section.

each segment are calculated. The minimum one is chosen as the true traveltime at the point P.

Actual Process

The actual procedure is shown in Figure 3. In this section, the procedure is illustrated using a numerical example. The specification of the numerical examples is shown in Table 1 and velocity model is shown in Figure 4. In this case, the model has two layers over a half-space. First of all, traveltimes corresponding to reflections from the same reflector are picked. Here, since the model has two reflectors, two datasets of traveltimes have been picked before applying this method. The method described in the previous section is applied layer by layer. The traveltime dataset of the first reflector have been processed and then the first layer is stripped to analyze the next layer. The two processes, kinematic migration and reflection tomography, are repeated until the estimated model converges, that is, the difference between observed traveltimes and calculated traveltimes becomes sufficiently small. The detailed procedure in the kinematic migration and the reflection tomography is as follows.

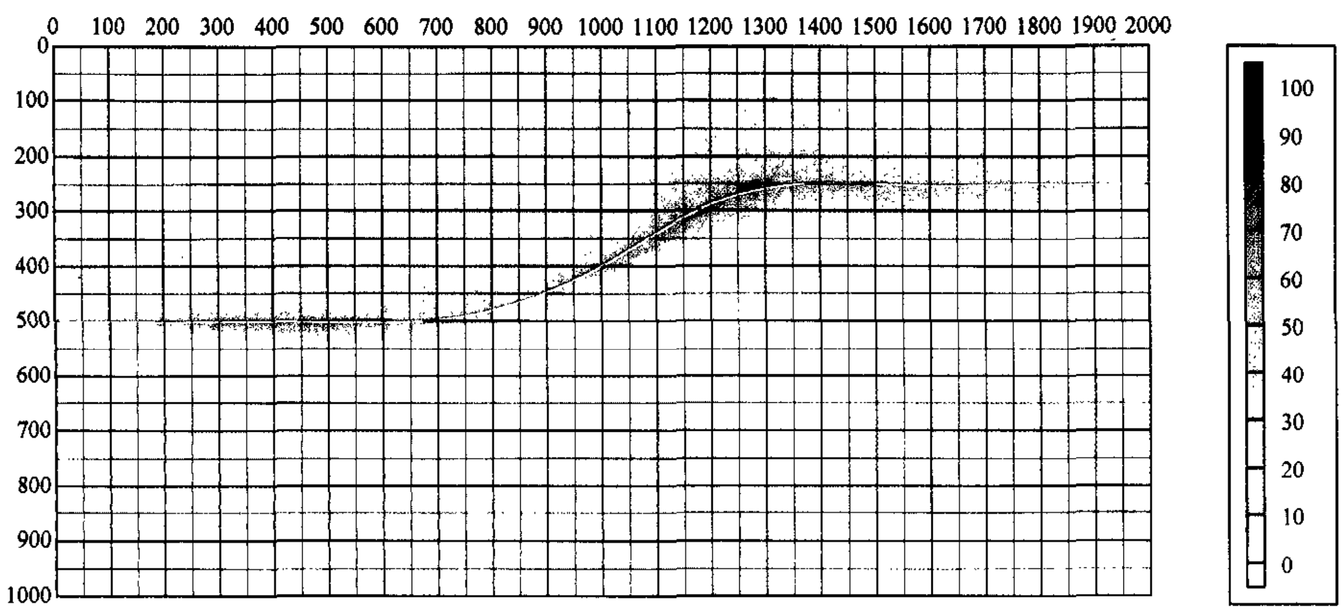

Fig. 7. The extracted reflector from the distribution map of the reflection points is shown with a solid line.

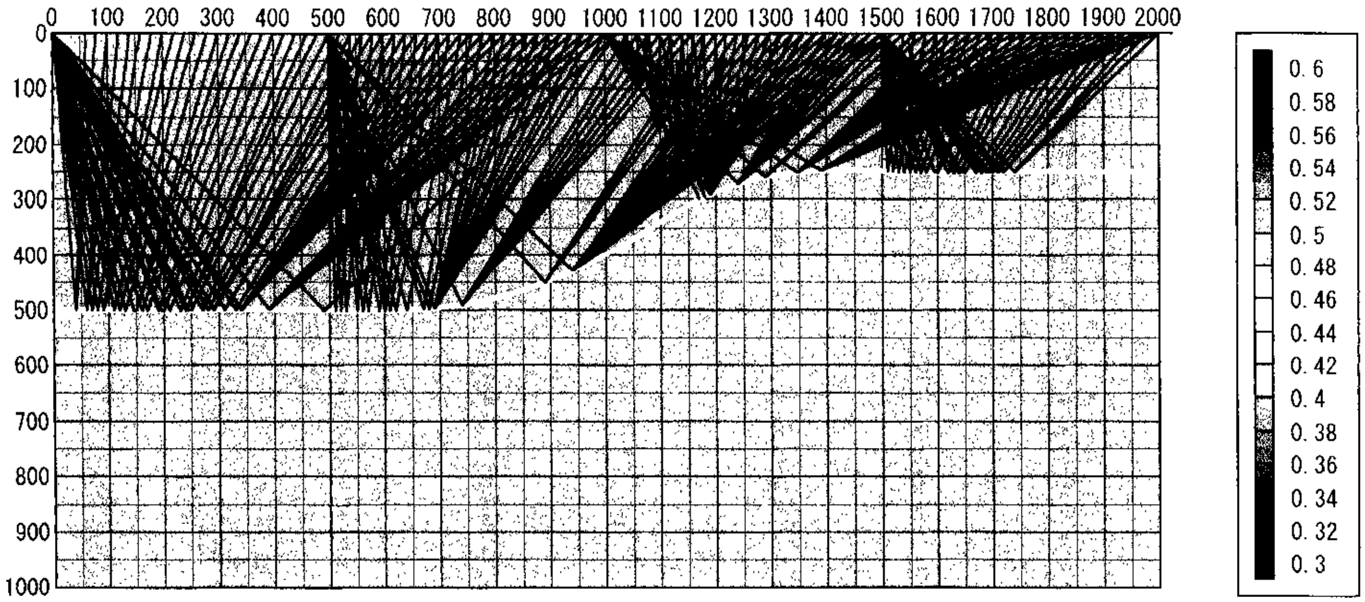

Fig. 8. Examples of raypaths reflected at the estimated reflector

Migration process

(1) A velocity model which consists of many cells is assumed as an initial model and is shown in Figure 5. The grid points are located on this velocity model independently from cell parameters. These points corresponds to the migration image points.

(2) The LTI forward process is applied to the velocity model and the traveltimes on grid points from all shot points and receiver points are computed using equation (4). The transmission traveltime maps on the grid points of all shot points and the reflection traveltime maps of all receiver points are obtained.

(3) An observed traveltime corresponding to a certain shot and receiver pair is processed as follows. A transmission traveltime map of the shot point and a reflection traveltime map of the receiver points are chosen. The two traveltime maps are summed in order to obtain a two-way traveltime map. According to the theory described above, an equi-two way traveltime curve is obtained by tracing grid points which have the two way traveltime equal to that of the observed traveltime. Since discrete gridded image points are used and the calculated traveltimes contains gridding errors, the points chosen are those which have two-way traveltime with some variations.

(4) Step (3) is repeated for every observed traveltime of the first layer. On every image point, the number of the

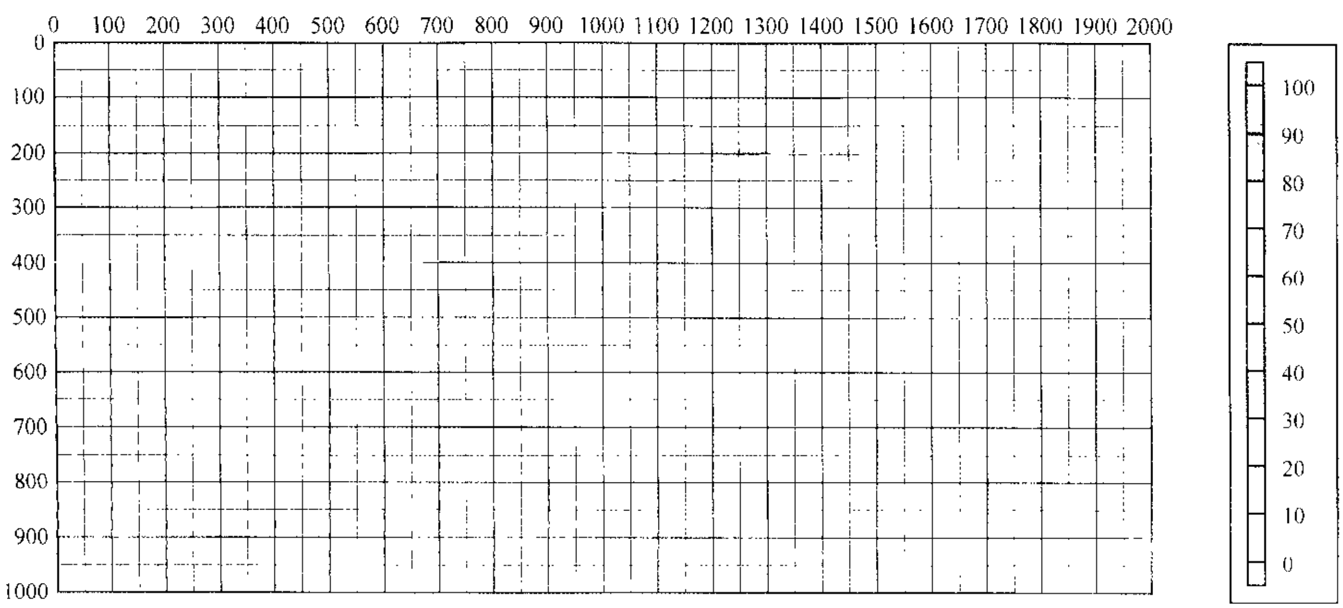

Fig. 9. The estimated reflector obtained after 10 iterations.

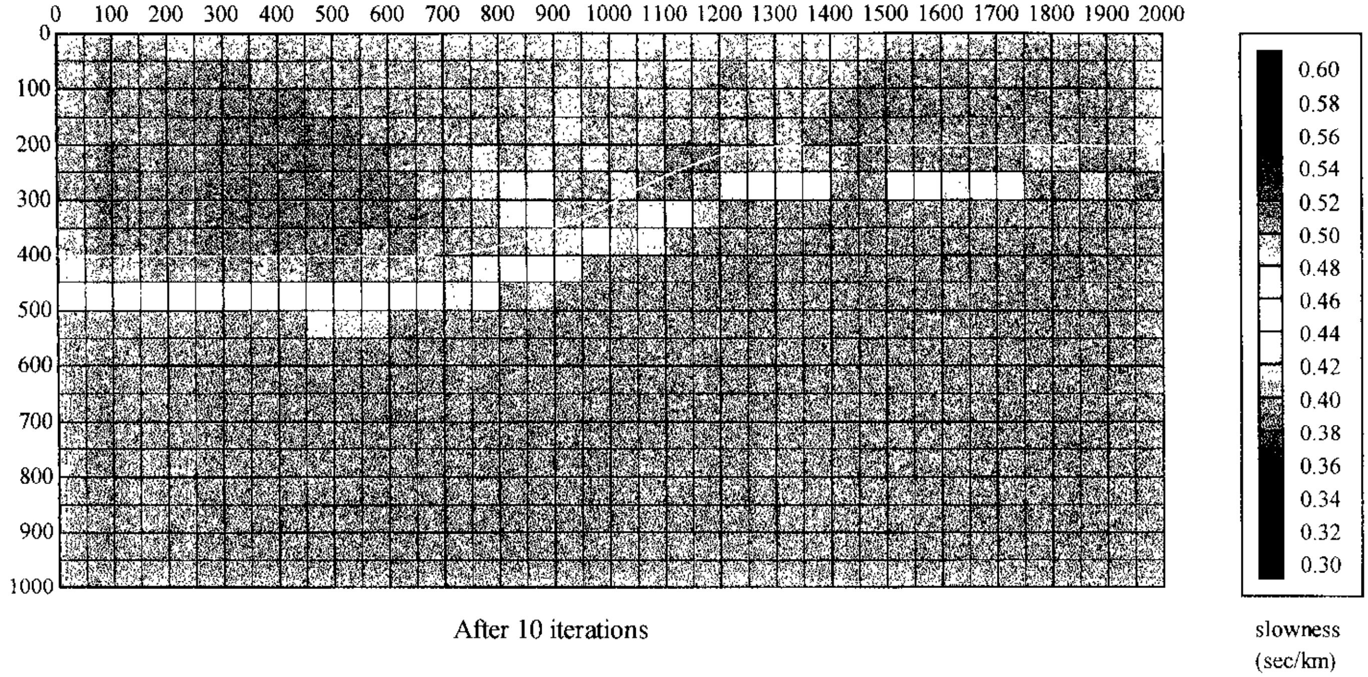

Fig. 10. The velocity structure obtained after 10 iterations.

equi-two way traveltime curves which passed on the image point is counted as processing the step (3).

(5) After completion of stage (4), the histogram of the equi-two way traveltime curve on the discrete image points in the depth section is obtained and represents the possibility of the reflection points associated with the first reflector. The results are shown in Figure 6.

(6) The depth of the reflector is estimated by picking the image points with the maximum values on the histogram at each horizontal position. However, the edge of the reflector cannot be estimated adequately because there are only a small number of reflection points. To overcome this, an extrapolation technique is applied at the edges. I note that because of using extrapolation, the reflectors need no longer lie on the grid points. The estimated reflector corresponding the first dataset of the traveltime is shown in Figure 7 by a solid line.

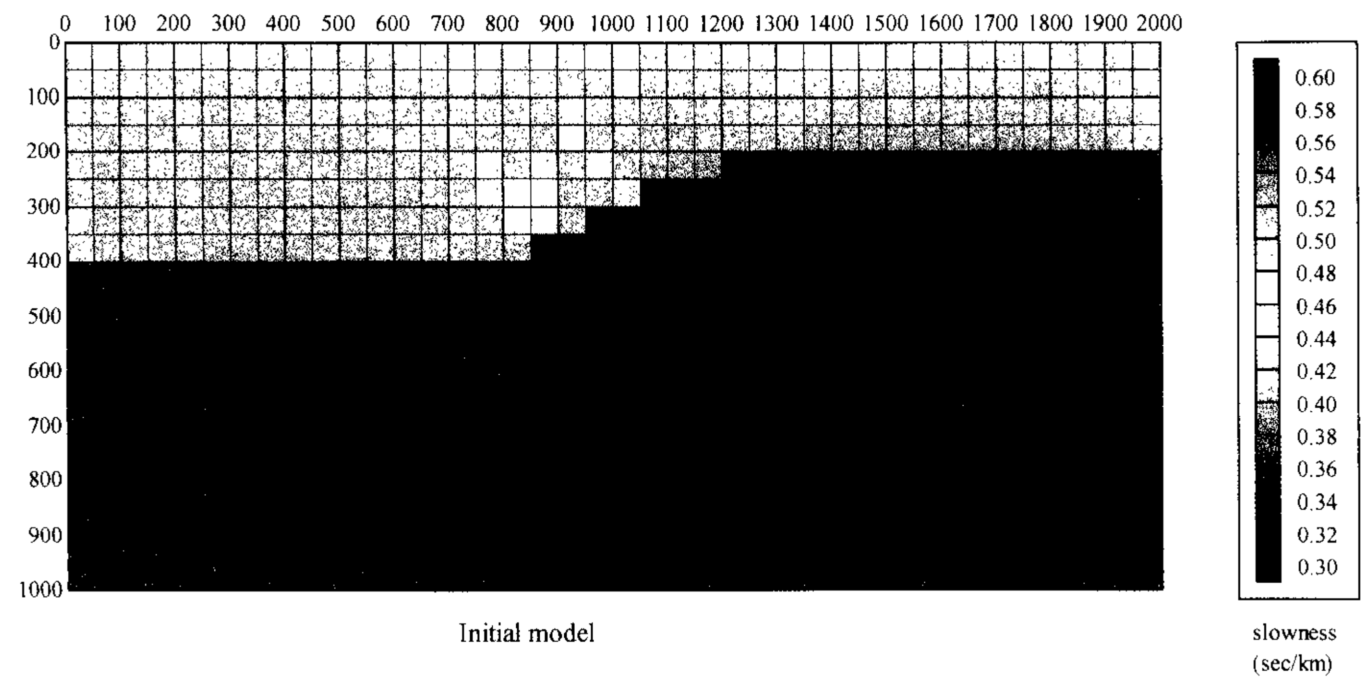

Fig. 11. Initial model for the second layer.

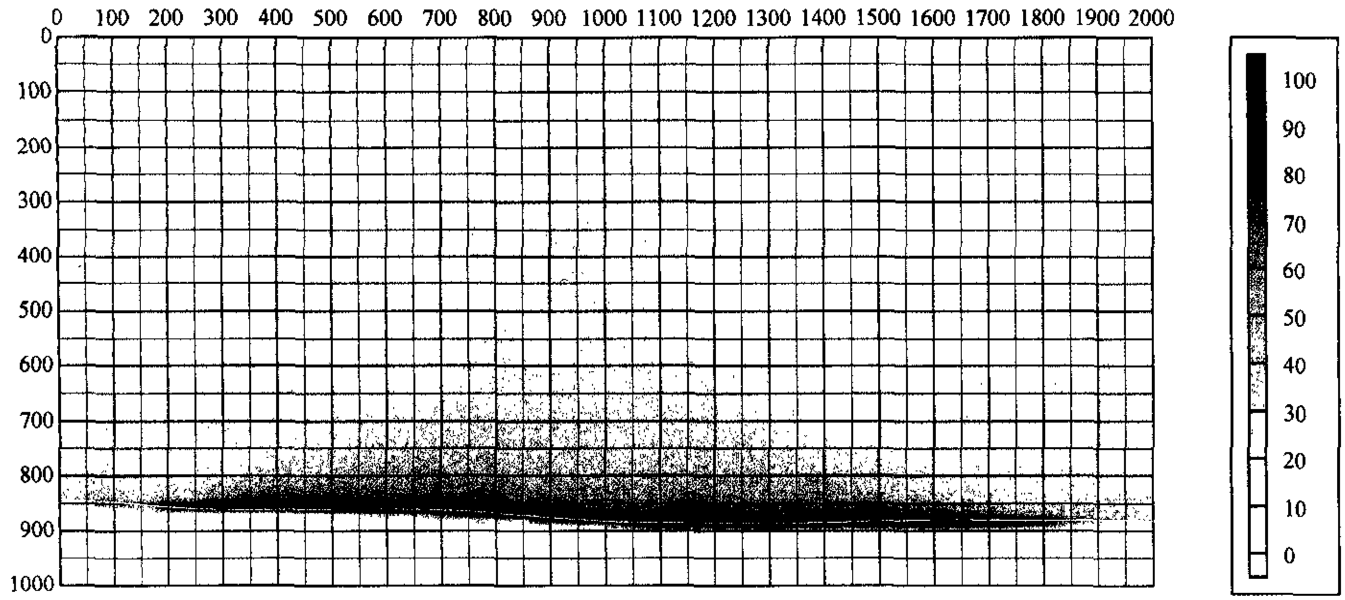

Fig. 12. The first estimation of the distribution of possible reflection points of the second layer.

Reflection tomography

(7) A raypath corresponding to a certain shot and receiver pair is calculated as follows. The first step is the estimation of the reflection point on the estimated reflector. A transmission traveltime map and a reflection traveltime map which correspond to the shot and receiver points, which have been calculated by LTI in the kinematic migration procedure, are chosen. Total traveltimes from shot and receiver point are calculated for every point of the estimated reflector using LTI equation (1). According to Matsuoka and Ezaka (1992), the reflection point is obtained by searching the point which has the minimum two way traveltime along the reflector. The LTI backward process is applied to the transmission traveltime map to get the transmission raypath from the shot to the reflection point. The LTI backward process is also applied to the reflection traveltime map to find the reflection raypath from the reflection point to the receiver point. By connecting the two raypaths, the raypath from the shot point to the receiver point through the reflection point on the reflector is calculated. One example of the raypath is shown in Figure 8.

(8) After the calculation of the raypaths for all pairs of shots and receivers, the observation equation for tomography analysis is obtained as

$$\mathbf{AS=T} \tag{6}$$

where **A** is the observation matrix. Each element, a_{ij}, is the

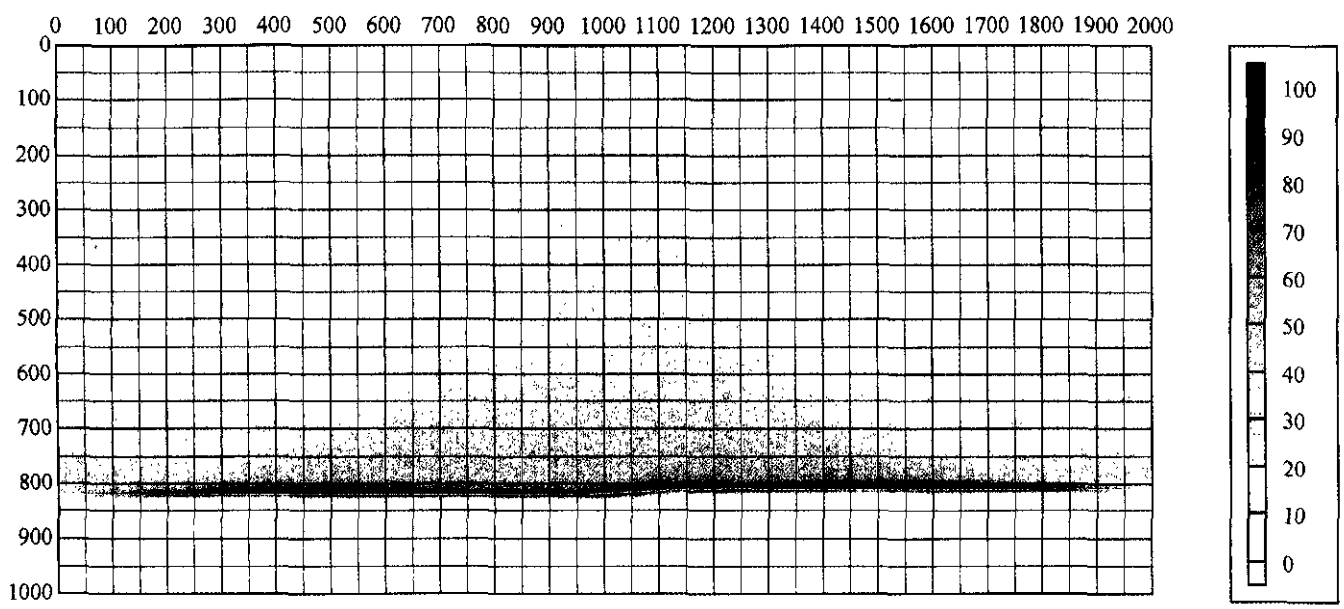

Fig. 13. The estimated second reflector after 10 iterations.

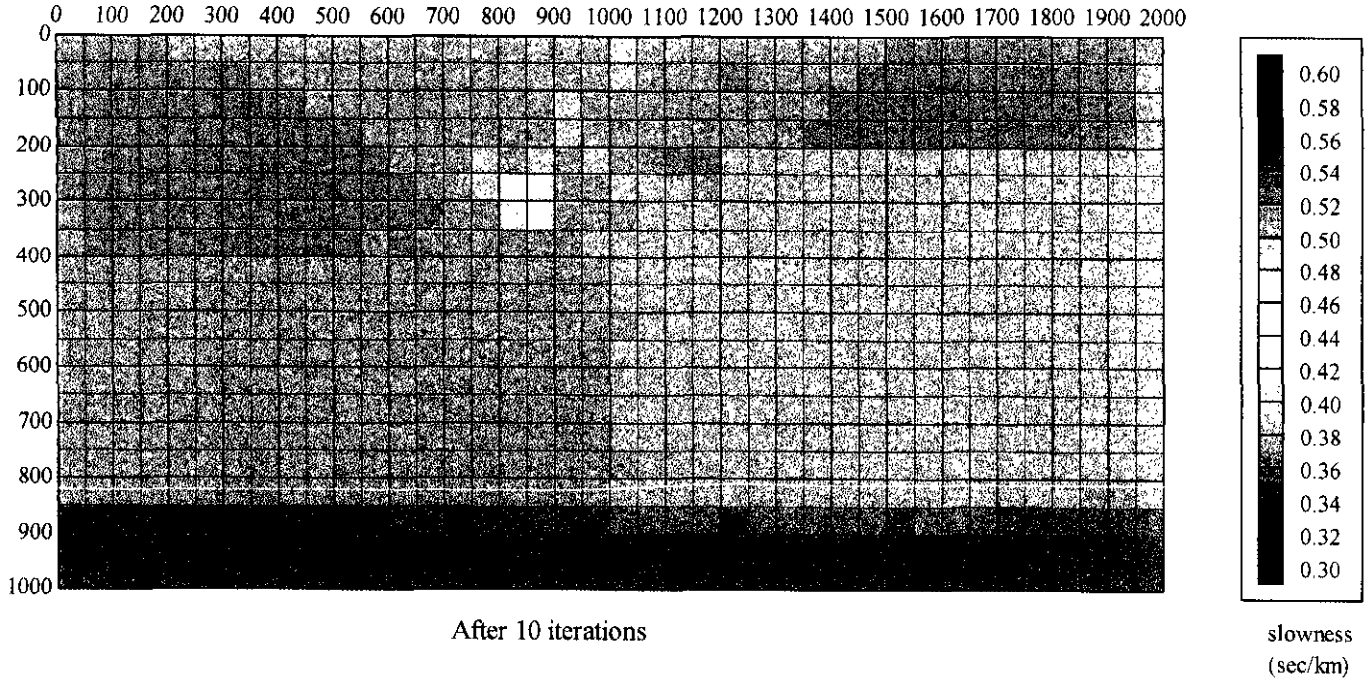

Fig. 14. The estimated velocity structure of the second layer after 10 iterations.

length of the i-th raypath traveling in the j-th cell. **S** is the column vector of slownesses, s_j, which is slowness in the j-th cell. **T** is the column vector of the traveltimes, t_i, which is the observed traveltime of the i-th raypath.

(9) In order to solve the observation equation in step (8), a backprojection technique is used. Solution of the observation equation gives the updated velocity (or slowness) structure above the estimated reflector.

(10) Then the kinematic migration process is applied again to the updated model. Steps (1)-(9) are repeated until the difference between calculated and observed traveltimes becomes small enough or the estimated model converges. Figure 9 shows the result of determining the reflector and Figure 10 shows the velocity distribution obtained after 10 iterations.

Iteration over the reflector

Once the first reflector and the velocity structure above the reflector are obtained, the next layer is determined under the condition that the first reflector and the velocity structure above the reflector are fixed. Constant velocity structure below the first reflector is assumed as shown in Figure 11. The application of kinematic migration process to the observed traveltime of the second reflector is shown in Figure 12. After the estimation of the second reflector, raytracing is performed as before and the observation equation same as equation (6) is obtained.

In order to update only the velocity below the first reflector, the observation equation is modified as follows. The observation matrix is partitioned into two parts which contain the elements above and below the first reflector. Specifically,

$$\mathbf{A}=\mathbf{Aa}+\mathbf{Ab} \qquad (7)$$

where **Aa** has the elements in the i-th cell above the reflector and **Ab** has the i-th in the cell below the reflector.

We now compute the traveltime vector **T1** in the first layer as

$$\mathbf{T_1}=\mathbf{AaS} \qquad (8)$$

where **S** is the estimated slowness in the first layer. The traveltime vector within the second layer, **T2**, is obtained by subtracting **T1** from the observed traveltime **T**. Finally the observation equation below the first reflector is obtained as

$$\mathbf{AbS}=\mathbf{T_2}. \qquad (9)$$

This equation is solved by backprojection and the updated velocity is obtained in the second layer.

Repeating the above procedure, the second reflector and the velocity structure in the second layer are obtained. The result for the second layer is shown in Figures 13-14. The final result is obtained by repeating the above steps layer by layer until all traveltime datasets are processed.

Discussion

In the numerical example described above, the first estimation of the background velocity, without any assumption concerning the position of the first reflector is 2500 m/sec. After step (6), the estimated depth of the reflector is nearly 1.25 times deeper than the true depth of 400 m. This first estimation is reasonable because the true velocity is 2000 m/sec and the traveltime is the product of depth and slowness. After reflection tomography is applied, the estimated velocity is slightly less than 2500 m/sec. This is because

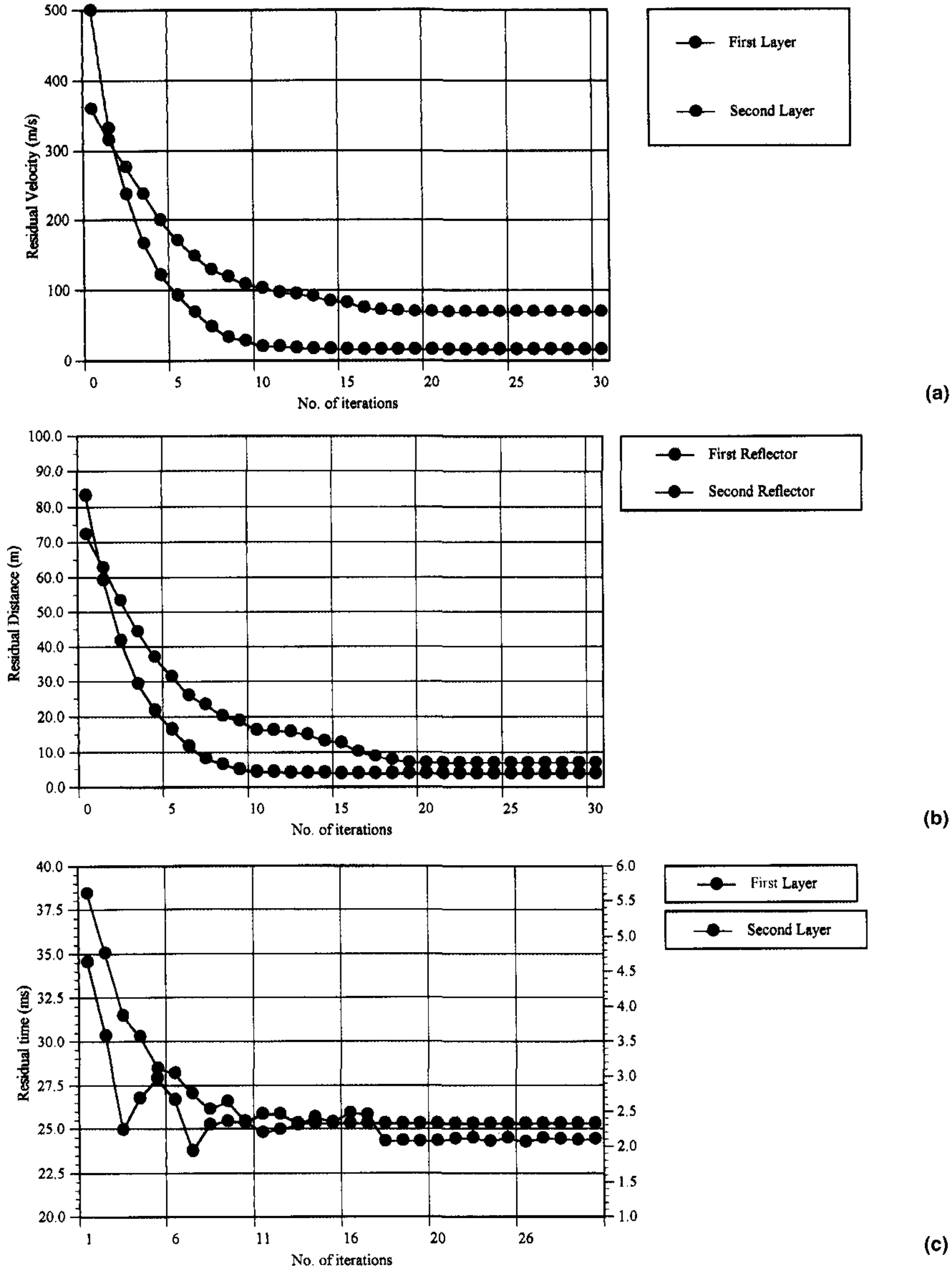

Fig. 15. Number of iterations vs. **(a)** residual velocity, **(b)** reflector position and **(c)** traveltime. The left y-axis indicates values of the first layer and right one indicates the second layer.

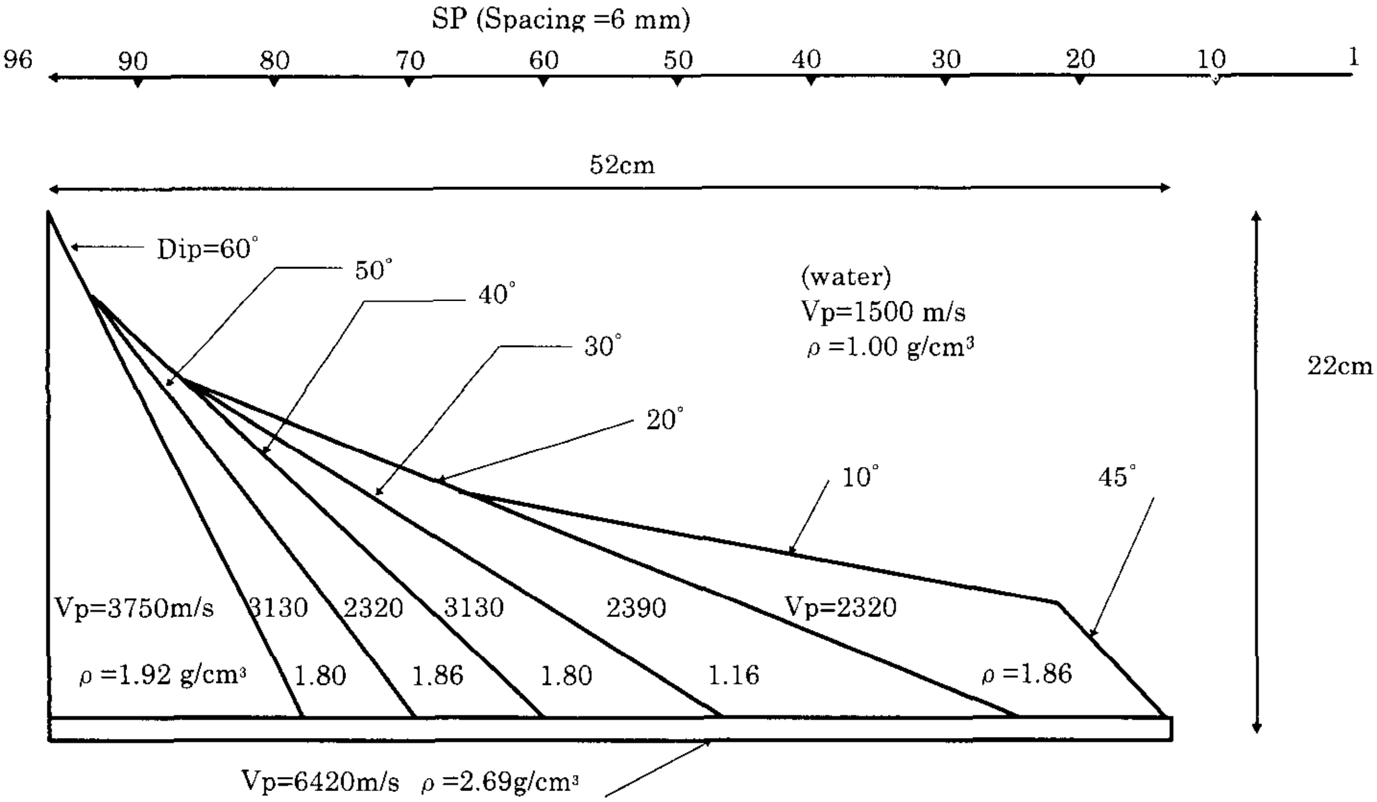

Fig. 16. Velocity structure of physical scale model.

there are many raypaths with different offsets. If the data are all zero-offset, the estimated velocity should be 2500 m/sec naturally and this method does not work well.

Since the velocity is estimated as less than 2500 m/sec, the second iteration of the migration process gives a slightly shallower depth than 500 m. Reflection tomography applied to the shallower reflector gives the lower velocity because the raypath is shorter and the traveltime is fixed. After iteration, the depth of the reflector becomes the true depth of 400 m and the velocity is 2000 m/sec. Once the true depth and velocity are obtained, further iteration does not change the depth or velocity. Figure 15(a)-(c) show the residuals of velocity, reflector position and traveltime when the number of iterations increases. In this example, ten iterations are enough to achieve convergence for the first layer.

Once the depth of the first reflector and the velocity above the reflector are estimated, the velocity below the reflector is assumed as 2850 m/sec, the true velocity being 2500 m/sec. The migration process is applied again to the assumed model and the estimated reflector recovered at a depth greater than the true depth of 800 m. Reflection tomography is applied to the estimated reflector under the condition that the velocity of the first layer is fixed. Then the estimated velocity is lower than 2850 m/sec. This process is repeated until the model converges.

Accuracy of the position of the second reflector is worse than that of the first reflector. The reason is that the number of the relatively large offset raypaths is small. Since two datasets of traveltimes are used in this example, two reflectors and velocities above the reflectors are obtained. If there are a number of traveltime datasets, they are processed layer by layer.

The most time consuming process in this method is the calculation of traveltimes using the LTI forward process. Since the results of the LTI forward process are used twice, once in migration and again in reflection tomography, the method presented in this paper is efficient.

Application to physical scale model data

This method is applied to actual dataset obtained in the physical scale model which consists of various epoxy-rubber. The top layer is water filled over the physical model. The exact velocity structure is shown in Figure 16. The dominant frequency of piezoelectric transmitter is 1 MHz and the scale factor is 10000. The specification of the dataset is shown in Table 2. The total number of traces is about 20000. The actual data of a common shot gather are shown in Figure 17.

In this physical scale model, there are 6 dipping reflectors over a half-space. It is difficult to pick up reflection events corresponding to all the reflectors. Instead, the first arrival and the traveltime reflected at the bottom of the model which are rather intense events are picked and analyzed. The reflection events corresponding to these events are shown in Figure 17. In some traces, it is difficult to determine the reflection events from the bottom because of low S/N ratios or multiples. Only the reflection events picked clearly are analyzed in this case.

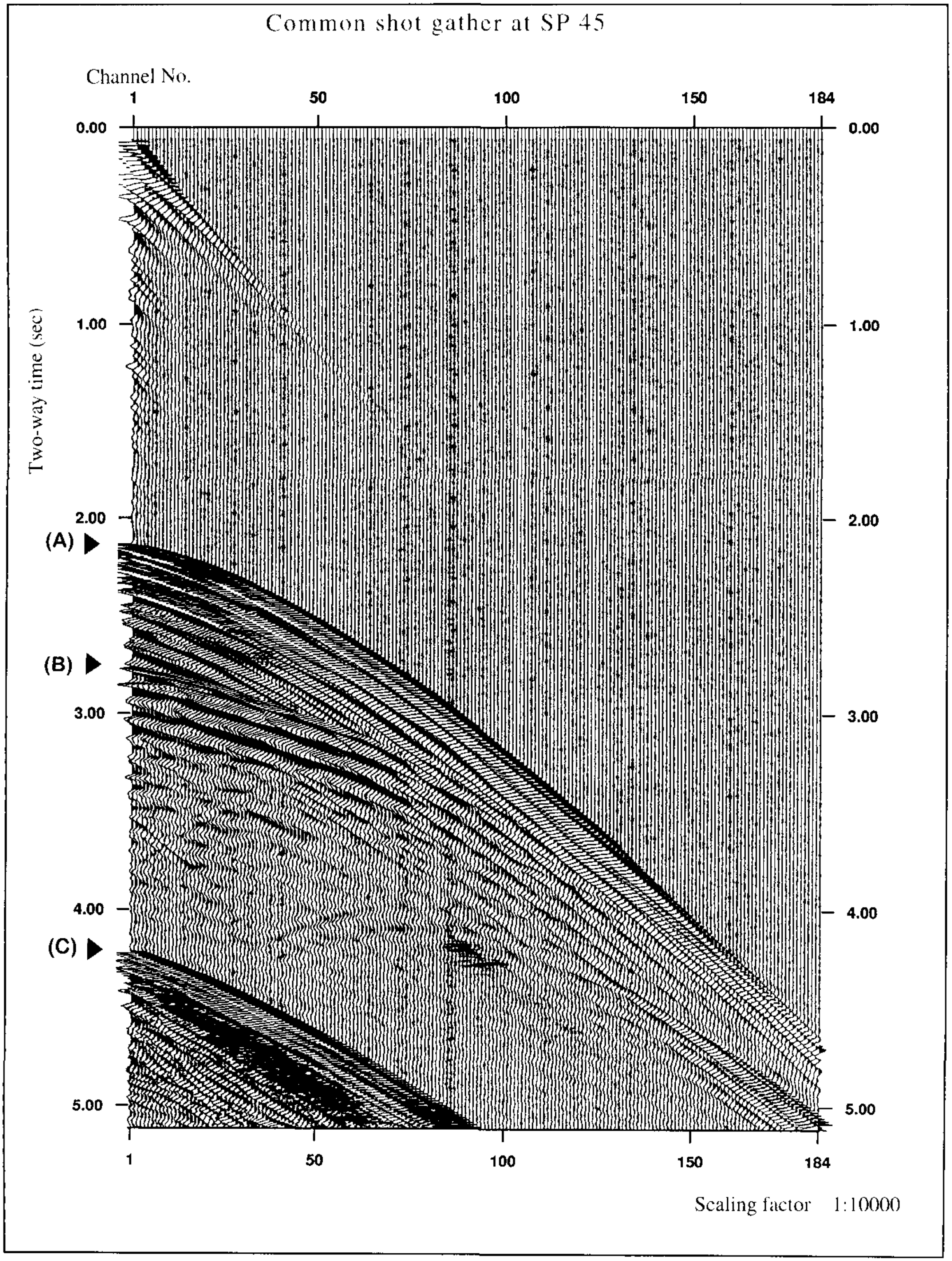

Fig. 17. Common shot gather obtained by scale model. Shot point (SP) is 45 (refer Fig. 16). (A) indicates the reflection events from the top of the model and (B) indicates the reflection events from the bottom. (C) indicates the multiple reflection events from the top.

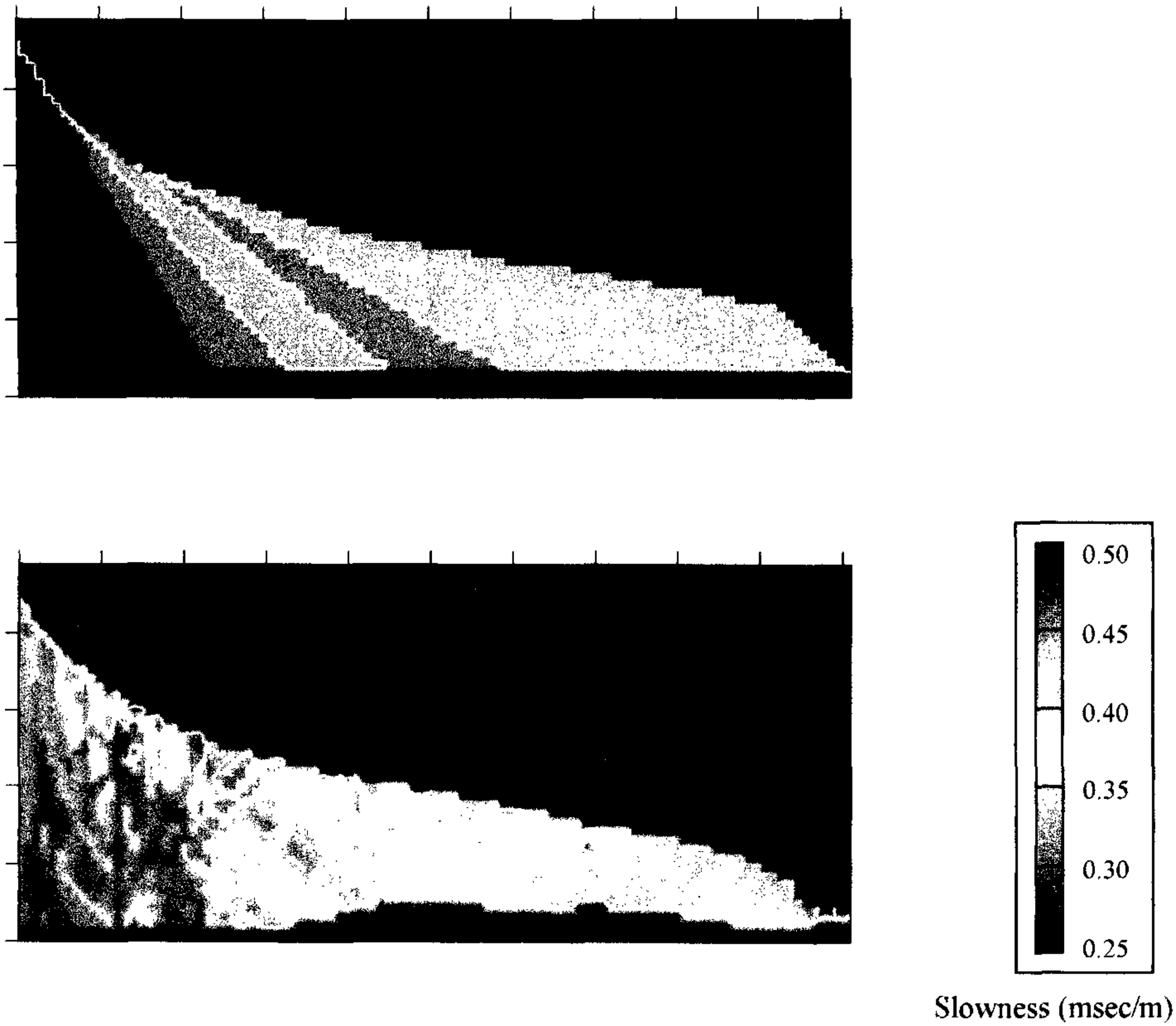

Fig. 18. The top figure shows the cell modeled velocity structure of the physical model. The bottom shows the final velocity structure estimated after 7 iterations.

The objective is to estimate velocity structure between the top reflector (water-model interface) and the bottom reflector. The position of the first reflector was estimated using the dataset of first arrival traveltime. In this case the exact velocity of water was given and only one iteration was needed to estimate the position of the reflector. After a position of the first reflector was obtained, the second dataset was analyzed. In this analysis, it required 7 iterations to converge. No limitation to the second reflector or velocity distribution was taken in this procedure. The initial model of the second reflector assumed constant velocity model (Vp=4.0km/s) below the estimated first interface. Figure 18 shows the final result. It shows the velocity variation in the second layer clearly. Reflection tomography uses nearly vertical raypaths and it is sensitive to lateral velocity variation, whereas cross-well tomography using nearly horizontal raypaths is suitable to resolve the vertical velocity variation.

In case of its application to the actual seismic data, this method is applicable to the datasets which are picked up only from the main reflectors. Tomographic analysis does not give the estimation of the sharp interface but gives the lateral velocity variations. So we can see the reflector interface variation on the tomographic result.

CONCLUSION

A new method to estimate both reflector position and velocity simultaneously is presented in this paper. This method is efficient and self-consistent, because it requires only traveltimes of reflection events of prestack seismic data.

The tomographic approach to estimate the velocity structure is effective in the case where the velocities vary horizontally, because it can handle the global velocity structure in the cell model that is more flexible than the layered model. However it has difficulties in estimating reflected raypaths and reflector position in the cell model. The proposed method overcomes the difficulties to estimate the reflected raypaths, interfaces and velocities in the cell model.

This method is applicable not only for seismic reflection tomography but also VSP or crosswell reflection tomography. In VSP especially, since controlled depth points in the well are available, a particularly accurate estimation of reflector depths and velocities is possible.

In practical application, the larger the number of raypaths which are used in the inversion process, the higher will be the resolution of the final image.

References

Asakawa, E. and Kawanaka, T., 1993, Seismic raytracing using Linear Traveltime Interpolation, Geophysical Prospecting, **41**, 99-111.

Bishop, T.N., Bube, K. P., Cutler, R.T., Langan, R.T., Love, P.L., Resnick, J.R., Shuey, R.T., Spindler, D.A. and Wyld, H.W., 1985, Tomographic determination of velocity and depth in laterally varying media, Geophysics, **50**, 903-923.

Matsuoka, T. and Ezaka, T., 1992, Raytracing using reciprocity, Geophysics, **57**, 326-333.

Stork, C. and Clayton, R.W., 1991, Linear Aspects of tomographic velocity analysis, Geophysics, **56**, 483-549.