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REFLECTION AND TRANSMISSION AT PLANE BOUNDARIES IN NONWELDED CONTACT

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ABSTRACT

Conventional seismic reflection and transmission theory assumes that the boundaries at an interface between two solids are in welded contact with each other. Described below is a theory of reflection and transmission by boundaries not in welded contact. The theory models each solid as a cubic crystal lattice with microscopic-sized particles at the lattice sites. The particles inside each solid and across the interface are connected to each other by springs representing Hooke's law forces. Only forces between nearest and next-nearest neighbours are considered. Nonwelded contact is introduced by making the springs across the interface sufficiently weaker than the springs inside the solids. Numerical examples are presented which show that nonwelded contact can substantially increase the strength of reflections, suggesting a possible partial explanation of anomalous seismic amplitudes reported occasionally in the literature. Furthermore, a planar fault in a single medium, which produces no reflected waves according to conventional theory because of the lack of an impedance contrast, can generate substantial reflected energy if the interface contact at the fault plane is nonwelded. An observation of reflected energy from such a fault plane in actual seismic data could hence be used to prove, or at least suggest, that nonwelded contact interfaces actually exist in the earth.

INTRODUCTION

Consider a planar fault in an infinite homogeneous medium (the material on one side of the fault plane is the same as that on the other side). Consider also a plane compressional wave incident upon the fault. Conventional seismic theory then says that there is no reflected compressional or shear wave because of the lack of an impedance contrast, *i.e.*, the reflection coefficients are zero. Conventional theory assumes that the boundaries on either side of the fault are in welded contact with each other, *i.e.*, the displacement and stress fields are continuous across the fault plane. However, if the two boundaries are not in perfect welded contact, reflected waves with appreciable amplitude *can* exist, even though there is no impedance contrast. In the more general case of two different solids separated by a plane interface, the reflection coefficients are also different from those calculated by conventional theory if the interface contact is loose or nonwelded. In the next section, a mathematical model of nonwelded contact is discussed. The model consists of representing the medium in a microscopic fashion: the two different solids on either side of the interface are modelled as cubic crystal lattices, with atomic-size particles occupying the lattice sites (the corners of the individual cubes). The particles are considered to be connected to each other by springs so that the motions of the particles (due to the passage of a wave) are governed by Hooke's law. Springs also connect the particles across the interface. Particle displacements are due to incident, reflected and transmitted waves, and Newton's second law of motion is applied to the particles to determine the reflection and transmission coefficients.

Such lattice models of wave propagation in solids have been used in solid state physics to calculate the atomic properties of solids (see, for example, Kittel, 1971). The various formulas so obtained often reduce to familiar macroscopic formulas in the long-wavelength limit (when the wavelength is much greater than the distance between adjacent particles in the lattice). In the nonwelded contact model discussed below, Newton's equations of motion for the particles on the boundaries of the two solids reduce, in the long-wavelength limit, to the familiar Zoeppritz equations relating the amplitudes of the reflected and transmitted waves, as long as the springs across the interface are strong enough (see next section). However, if they are not, the amplitude equations differ from Zoeppritz's equations even for long wavelengths, resulting in different reflection and transmission coefficients (the case of nonwelded contact). It is not inconceivable that interfaces in the Earth may exist at which the contact between the boundaries is not

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I thank Prof. F. Hron for the use of his graph-plotting program which I have modified for the purposes of this paper. I take this opportunity to correct a typographical error in the paper by Paranjape *et al.* (1987): in equation 31, $\bar{k}'$ should be k'.

perfect. It is perhaps possible that some anomalous amplitudes in seismic data may be the result of an imperfect nonwelded contact at a reflecting interface.

Model and Theory

As mentioned above, each of the two solids is represented by a cubic crystal lattice with lattice constant a (the distance between a particle and its nearest neighbour). For simplicity, a is assumed to be the same for both solids, and a is also the distance across the interface. The parameters α_1 and α_2 are the force constants of the springs connecting nearest neighbours in solids 1 and 2, respectively. Similarly, β_1 and β_2 are the force constants of the springs between the next-nearest neighbours. α and β are the force constants of the springs connecting nearest and next-nearest neighbours across the interface (see Figure 1). In each solid, the particle masses are all the same, although the particle mass in solid 1 is different from that in solid 2. In applying Newton's second law of motion (force = mass x acceleration) to a given particle, only the displacements of the particle's nearest and next-nearest neighbours are considered to contribute to the force acting on the particle.

The mathematics of the theory is presented in detail by Paranjape *et al.* (1987) and is only briefly described in what follows. A plane harmonic compressional wave is allowed to impinge on the interface, generating reflected and transmitted compressional and shear waves. In general, each wave will have both compressional and shear components, but to keep things relatively simple, this complication is not introduced. This, however, leads to the restrictions $\alpha_1 = 2\beta_1$, $\alpha_2 = 2\beta_2$, and that the compressional/shear-wave speed ratio in each solid is 3 (which is equivalent to a Poisson ratio of 0.25 in each solid). Only under these conditions can purely compressional and shear waves propagate in the lattices when only nearest and next-nearest neighbour interactions are considered. Hence, some generality is lost. However, this is not a serious drawback, since the intention here is to obtain some more or less qualitative results on the effect of nonwelded contact rather than accurate estimates of the scattered-wave amplitudes in any arbitrary case. To obtain the relative amplitudes of the scattered waves, Newton's second law is applied to a given particle on the boundary of side 1 and also to the corresponding particle on the boundary of side 2, resulting in four equations of motion in the horizontal and vertical components of displacement. The displacement field on the incidence side (side 1) is the sum of the incident, reflected compressional, and reflected shear waves. The field on side 2 is the sum of the transmitted compressional and shear waves. The formulas for the displacements are substituted into the four equations of motion, giving four equations in four unknowns (the four scattered-wave amplitudes). The condition that the amplitudes cannot depend explicitly on position along the interface leads to Snell's law, as in the macroscopic case. The condition $\alpha = 2\beta$ is also substituted for simplicity. A long-wavelength approximation is made since the wavelengths of interest are much greater than the lattice constant a. Through various manipulations, these four equations for the scattered-wave amplitudes can then be made to look just like the familiar Zoeppritz equations, except for extra terms that appear in the coefficients of the unknowns. Letting k be the wavenumber of the incident compressional wave ($k = 2\pi/\lambda$ where λ is the wavelength), each extra term can be written so that it is proportional either to the very small number ka or to η, where

$$\eta = \frac{\alpha_1 ka}{\alpha} \tag{1}$$

(it is assumed that α_2 is roughly of the same order of magnitude as α_1, see equation (2) below). Hence the four equations reduce to the Zoeppritz equations in the long-wavelength macroscopic limit ($ka \rightarrow 0$), as long as $\alpha/\alpha_1 >> ka$. However, the parameter η shows that if α/α_1 is about as small as ka, then $\eta \sim 1$ and the extra terms containing η are no longer small, even for long wavelengths. This means that the solutions of the four equations will be significantly different from those of the conventional Zoeppritz equations. In other words, if the "bonds" (springs) across the interface are sufficiently weaker than the bonds between the particles inside the solids, the amplitudes of the scattered waves can be significantly different from those calculated by conventional elastic theory, even for long wavelengths. This is the case of nonwelded contact at the interface.

Note that the interface bonds do not have to be strong in the absolute sense in order for the four equations of motion to reduce to the familiar Zoeppritz equations for welded contact, *i.e.*, the interface bonds can still be much weaker than the bonds inside the solids: α/α_1 and ka can both be very small, as long as $\alpha/\alpha_1 >> ka$ so that $\eta << 1$.

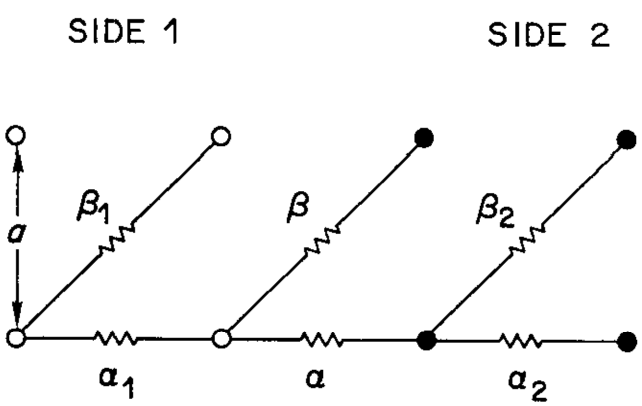

Fig. 1. Lattice model of two solids in contact. Some of the Hooke's law forces between particles of the solids are indicated by springs and their corresponding force constants. In each solid, the distance between nearest neighbours is a, which is also the distance across the interface.

Examples and Discussion

It is helpful to look at numerical values of the above-mentioned parameters. If the particles at the lattice corners are considered to be atomic-sized, then $a \sim 10^{-10}$ m. A typical value for the wavelength in exploration seismology is $\lambda \sim 60$ m. This gives $ka \sim 10^{-11}$. The effects of nonwelded contact occur for $\eta \sim 1$, which implies $\alpha/\alpha_1 \sim 10^{-11}$. In other words, the interface-solid bond-strength ratio would have to be very small in this model for the nonwelded contact effects to be significant. The word "small", however, is somewhat arbitrary. For instance, if α/α_1 would be changed to 10^{-7}, the situation would move from nonwelded contact to welded contact, even though 10^{-7} is also a very small number (see the last paragraph of the previous section). Hence, one should not attach a great deal of physical significance to the smallness of α/α_1. Furthermore, in carrying out the computations to solve the equations for the reflection and transmission coefficients, it is not necessary to use a realistic value such as 10^{-11}, *i.e.*, setting $\alpha/\alpha_1 = ka = 10^{-5}$, say, for $\eta = 1$, yields the same numerical results as $\alpha/\alpha_1 = ka = 10^{-11}$. It is the ratio of these two quantities, *i.e.*, η, that is the determining factor, not the quantities themselves. This is another indication of the arbitrariness of the values of ka and α/α_1 and also suggests that one does not really need to consider the particles at the lattice sites as atomic-sized particles. One may think of the medium as being modelled by a "fictitious" lattice of "microscopic-sized" particles (*e.g.*, a could be 10^{-4} m). Once the final equations for the nonwelded contact case have been derived, those equations can then represent the nonwelded contact model, and the physical significance of the lattice structure used to conveniently derive the equations can be de-emphasized.

From the theory, one may easily derive the following equation relating the microscopic force constants α_1 and α_2 to the macroscopic densities ρ_1 and ρ_2 and compressional wave velocities v_{P1} and v_{P2} in the two media:

$$\frac{\alpha_2}{\alpha_1} = \frac{\rho_2}{\rho_1}\left(\frac{v_{P2}}{v_{P1}}\right)^2. \tag{2}$$

For instance, if $\rho_1 = \rho_2$ and $v_{P2}/v_{P1} = 2$, then $\alpha_2/\alpha_1 = 4$. To apply the theory in a macroscopic situation, *i.e.*, to compute reflection and transmission coefficients in a welded contact or nonwelded contact case, one need only input values for the densities and compressional-wave velocities (actually, the density and velocity contrasts are sufficient) and a value for η. (It is assumed that ka is set to a value much less than 1 in the computations.)

Figures 2 to 5 show relative reflection and transmission amplitudes as a function of the angle of incidence of the compressional wave. The figures show the solutions of both the conventional Zoeppritz equations for perfect welded contact and the four equations for the nonwelded contact case discussed in the previous section. The parameter η is a measure of the degree of looseness or nonwelded contact at the interface and has the value 1 in Figures 2 to 5. Figure 2 indicates that, at normal incidence, the magnitude of the reflected compressional wave for the nonwelded contact case is about three times that of the wave in the conventional welded contact case. Furthermore, there is a phase difference of more than 90 degrees, so that not only the magnitude but also the phase of the recorded waveform would be substantially different. Figure 3 indicates that the reflected shear wave has a significantly higher magnitude for most angles of incidence in the nonwelded contact case also. The differences between the two cases are not as great, however, for the transmitted waves. If we would let η approach zero, the nonwelded contact curves would approach those for welded contact. The differences between the welded and nonwelded contact cases are basically independent of the medium parameters (impedance contrast) but, of course, depend strongly on η.

Figures 6 to 9 show the relative amplitudes in the case of a planar fault in a single solid, *i.e.*, sides 1 and 2 consist of the same material. The value of η is 1 for the nonwelded contact curves. In this case, conventional welded contact theory states that since there is no impedance contrast, there can be no reflected waves, and that the incident wave is merely transmitted through the interface without change. This is indicated in the figures. However, if the boundaries at the interface are not in welded contact, there can be a substantial amount of energy reflected. In particular, as Figure 6 indicates, the magnitude of the reflected compressional wave for normal incidence can be relatively high. As before, there is not as much of a difference between the welded contact and nonwelded contact cases for the transmitted waves as for the reflected waves.

Figures 10 to 13 are the same as Figures 6 to 9 except that $\eta = 0.5$ instead of 1. As expected, the nonwelded contact curves are closer to the welded contact curves and would coincide with them for $\eta \rightarrow 0$.

In recent years, seismic interpreters have placed more emphasis on relative amplitude analysis (including magnitude and phase) than in the past. Various factors affecting relative amplitude, such as geometrical spreading of wavefronts, absorption, offset, etc., are being dealt with more carefully. The above examples show that nonwelded contact is another factor that can possibly be added to the list. I do not know whether or not interfaces exhibiting nonwelded contact actually exist in the earth, but, in my view, it seems possible that they could. Neither do I know whether or not the theoretical model discussed above gives an adequate description of the effects of nonwelded contact (if it exists) on waves in the earth. Perhaps such questions could be answered by detailed analyses of appropriate seismic data (which I leave for future work). In the case of two different solids joined at an interface, careful measurements of

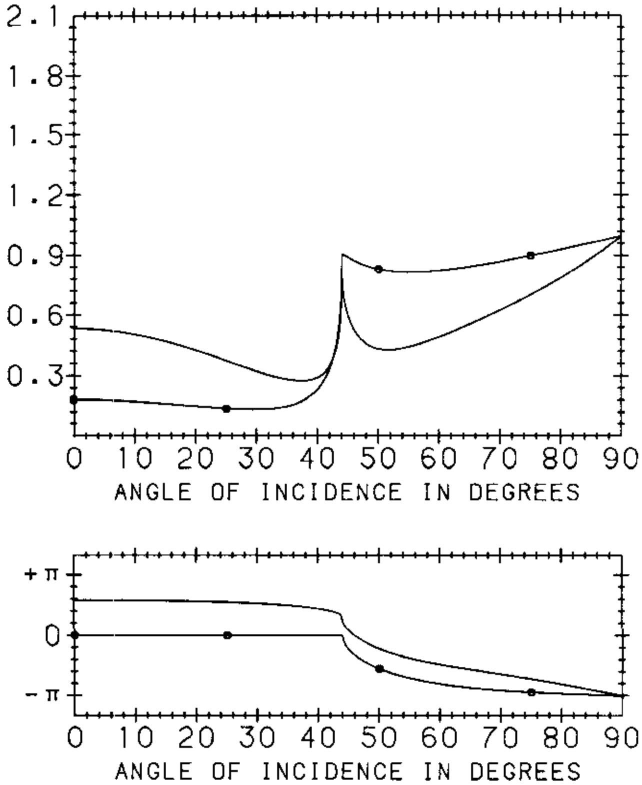

Fig. 2. The relative magnitude (upper graph) and relative phase (lower graph) of the displacement for the reflected compressional wave (sometimes called a reflection coefficient). The density is the same in both media. The ratio of the compressional-wave speed in the transmission medium to that in the incidence medium is 1.44. Poisson's ratio is ¼ in both media. The curves with octagonal markers are the solutions of the conventional Zoeppritz equations for perfect welded contact, and the curves without markers are for nonwelded contact, with $\eta = 1$.

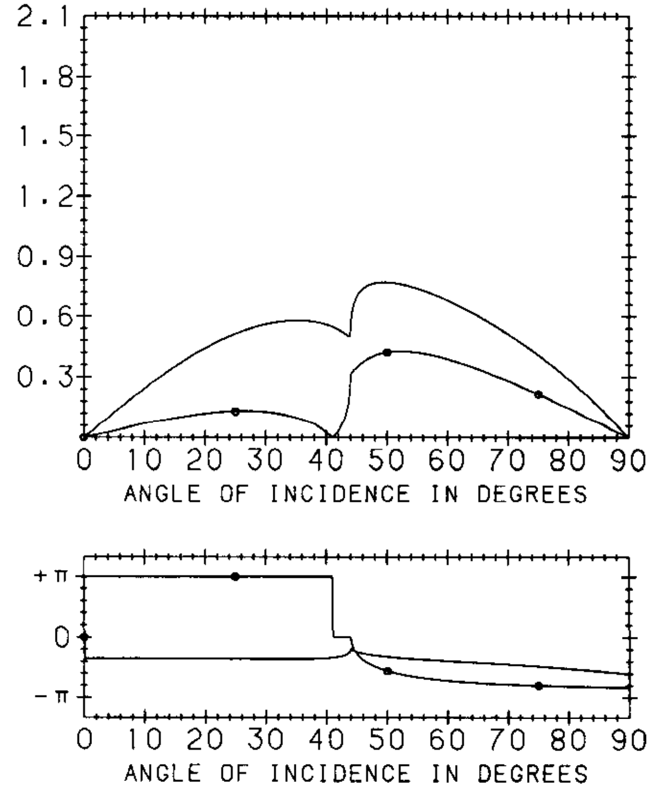

Fig. 3. Relative magnitude/phase of the reflected shear wave. For details, see the caption of Figure 2.

magnitude and phase versus offset or angle of incidence, together with density and velocity information, and combined with theoretical calculations using the conventional and nonwelded contact theories discussed above, might permit one to decide whether or not a given interface exhibits nonwelded contact. A better approach might be to make magnitude/phase measurements on reflections from a fault in a single solid, since then any nonzero reflected energy measured would indicate the presence of nonwelded contact.

Another factor which can cause reflection and transmission coefficients to deviate from their conventional values is anelasticity (see, for example, Krebes, 1984). This raises the question of determining whether an amplitude anomaly is due to anelasticity or to nonwelded contact. As shown by Krebes (1984), anelastic reflection and transmission coefficients differ very little from elastic ones for small angles of incidence (although differences can be great for larger angles), whereas as shown by Figure 2 of this paper, the difference between the welded contact and nonwelded contact compressional-wave reflection ceofficient can be relatively large at small angles of incidence. This would provide a means of distinguishing between the effects of anelasticity and

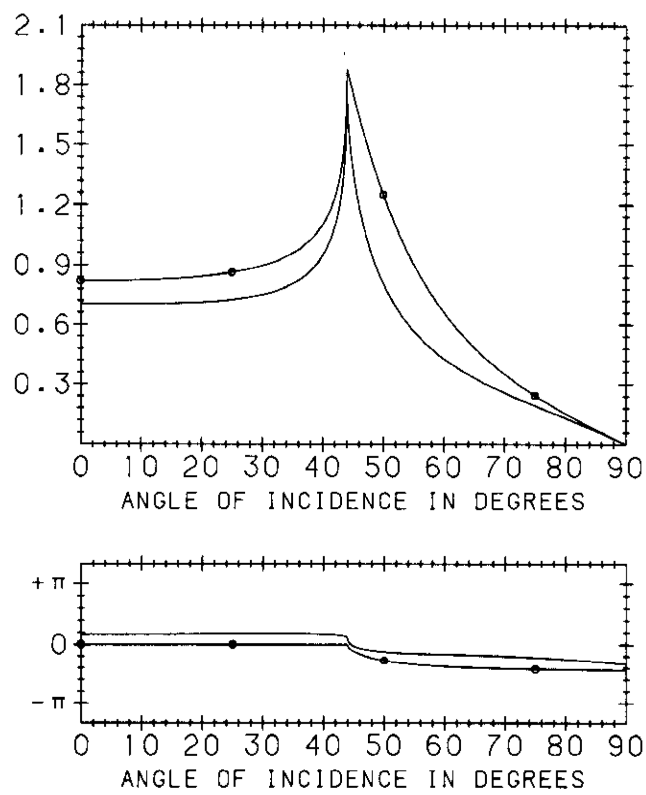

Fig. 4. Relative magnitude/phase of the transmitted compressional wave. For details, see the caption of Figure 2.

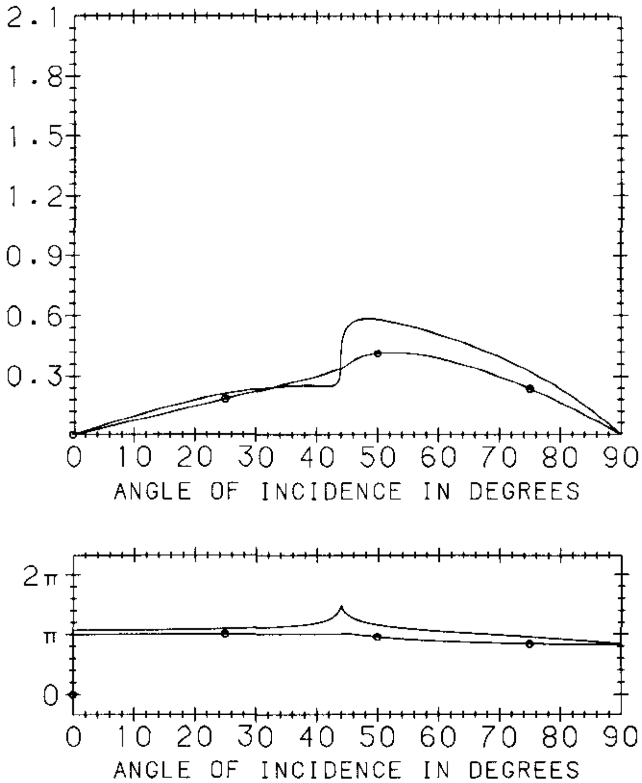

Fig. 5. Relative magnitude/phase of the transmitted shear wave. For details, see the caption of Figure 2.

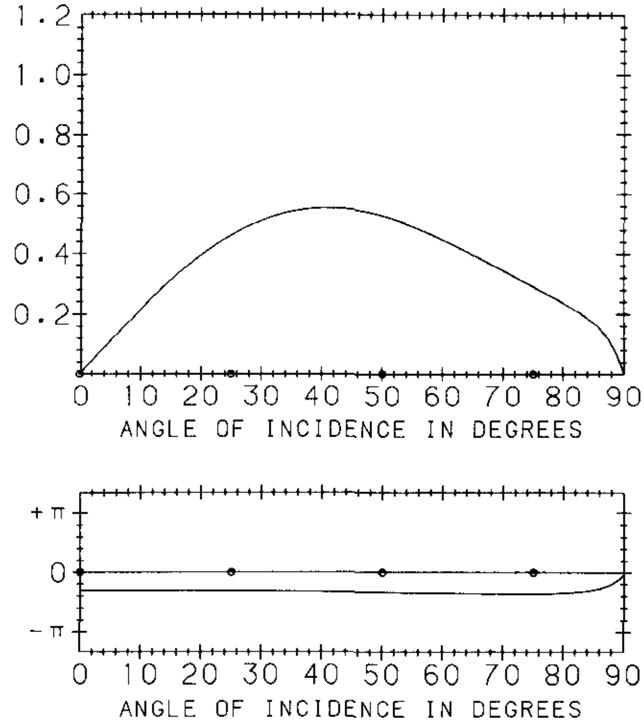

Fig. 7. Relative magnitude/phase of the reflected shear wave. For details, see the caption of Figure 6.

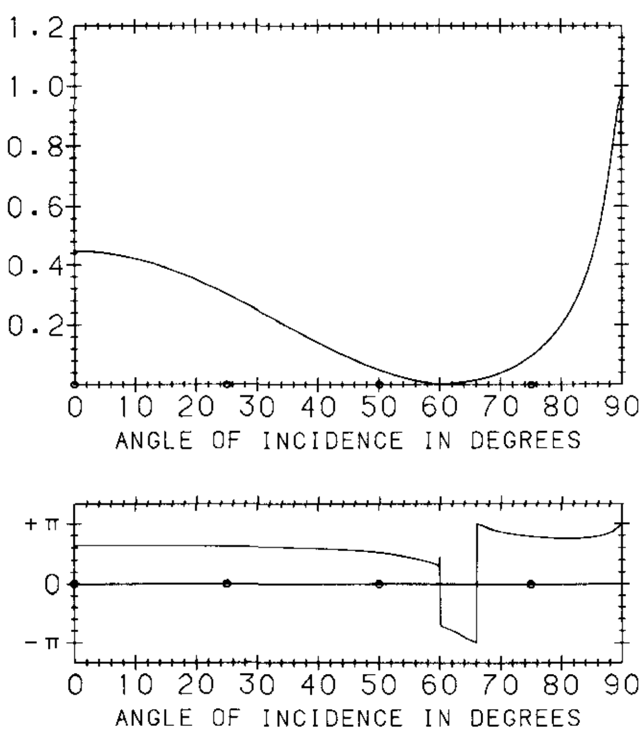

Fig. 6. The relative magnitude and relative phase of the displacement for the reflected compressional wave for a planar fault in a single homogeneous medium (any density and wave speed). Poisson's ratio is ¼. The curves with and without octagonal markers are for welded and nonwelded contact, respectively, and $\eta = 1$.

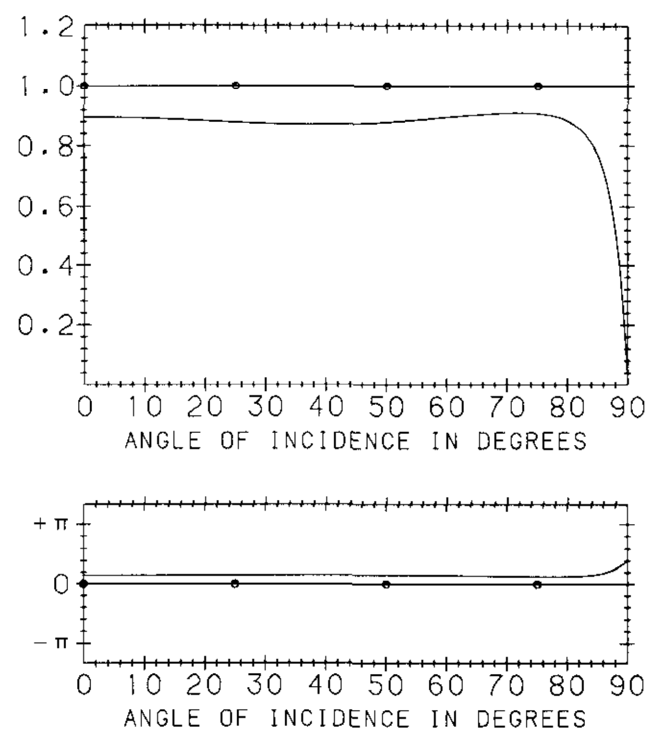

Fig. 8. Relative magnitude/phase of the transmitted compressional wave. For details, see the caption of Figure 6.

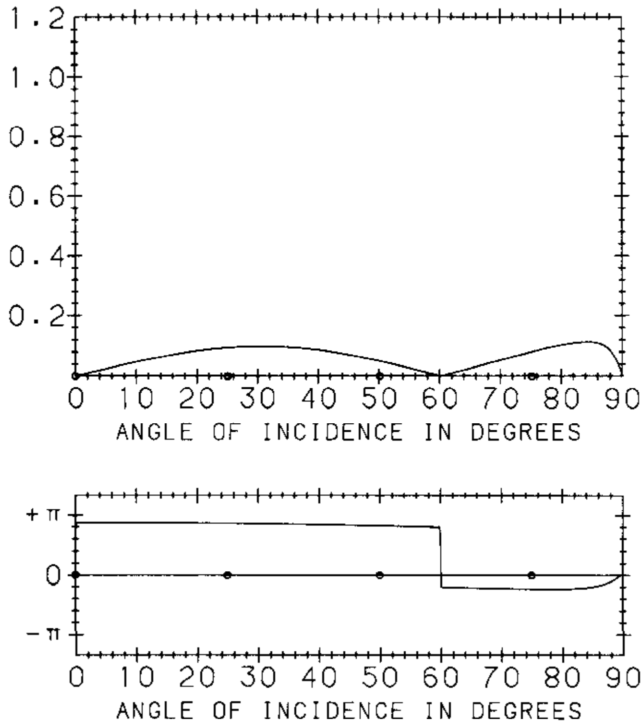

Fig. 9. Relative magnitude/phase of the transmitted shear wave. For details, see the caption of Figure 6.

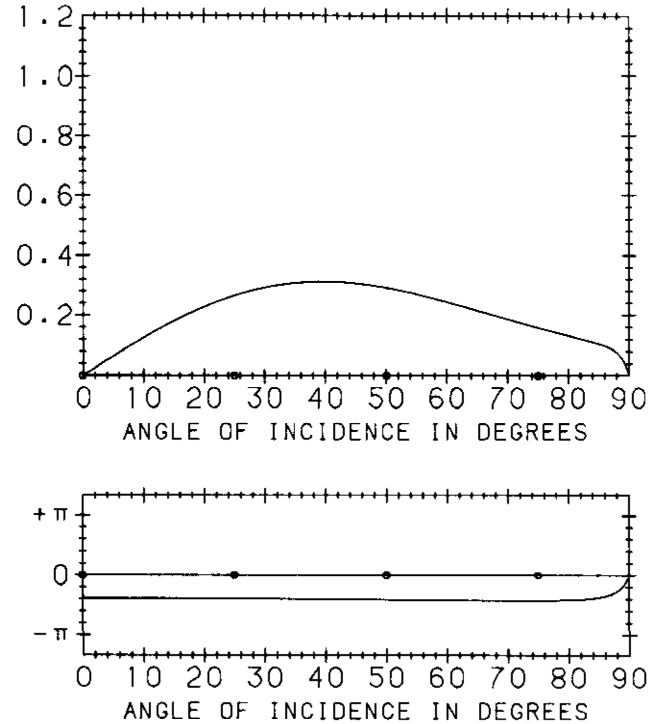

Fig. 11. Same as Figure 7, except $\eta = 0.5$.

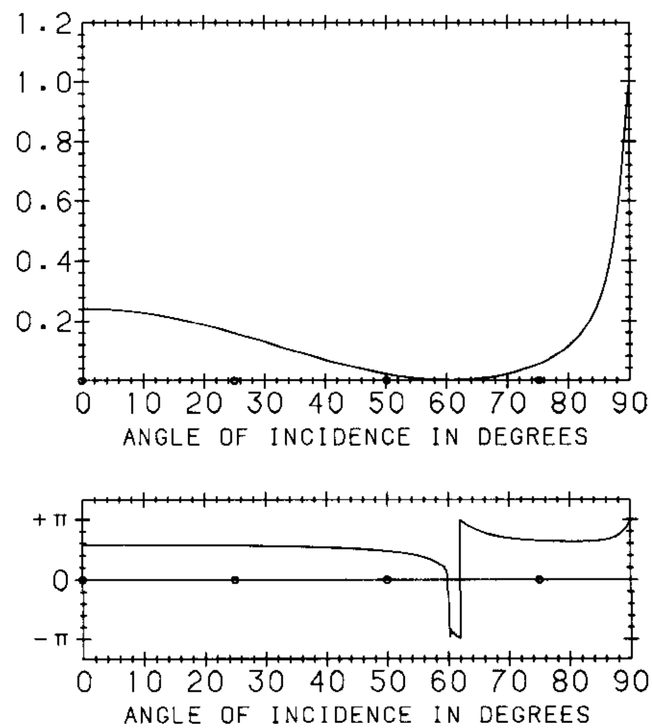

Fig. 10. Same as Figure 6, except $\eta = 0.5$.

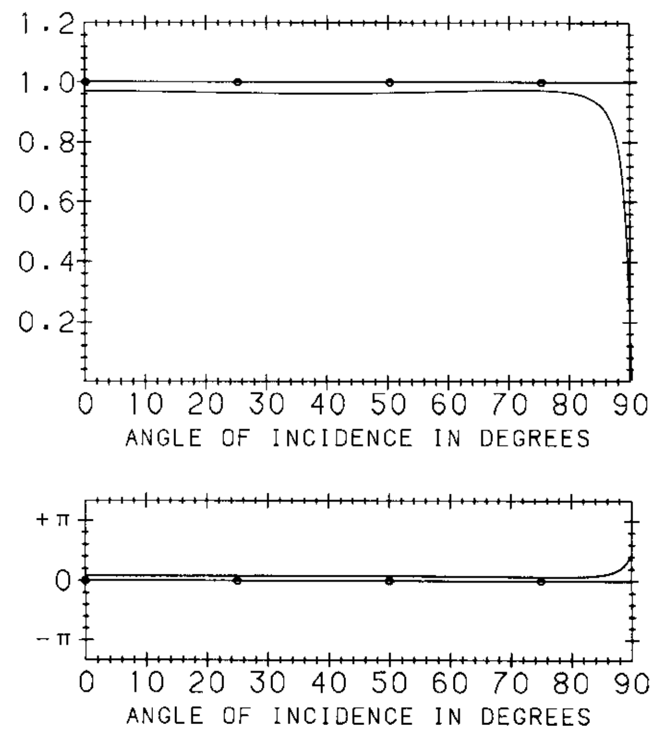

Fig. 12. Same as Figure 8, except $\eta = 0.5$.

nonwelded contact on amplitude. Also, nonzero reflections from faults in a single solid would signify nonwelded contact, as mentioned in the previous paragraph, since the presence of anelasticity cannot produce nonzero reflections from such faults (because of the lack of an impedance contrast). In general, for larger angles of incidence and for situations in which the two effects cannot be separated easily, and when other factors which affect amplitude, such as anisotropy, are present, one would likely have to use theoretical modelling to

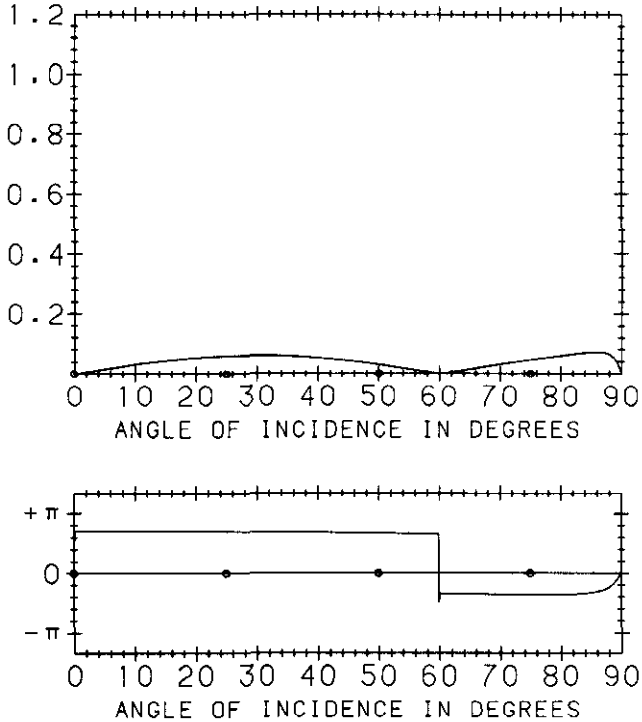

Fig. 13. Same as Figure 9, except $\eta = 0.5$.

distinguish all the effects. Synthetic seismograms accounting for all the physical factors (nonwelded contact, anelasticity, anisotropy, etc.) would be helpful in the general case.

It seems possible that various seismic amplitude anomalies reported from time to time in the literature may be explained, at lease partially, by reflections from boundaries in nonwelded contact. To cite just one example, Hurich *et al.* (1985) reported unexpectedly large reflection amplitudes from mylonite zones where acoustic contrasts are small. Although they suggested, as the cause, constructive interference due to layering, it is not unreasonable to consider the possibility of nonwelded contact playing a role. Such an hypothesis would have to be thoroughly tested, however, before any firm conclusions could be drawn.

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