

## FIRST-BREAK INTERPRETATION USING GENERALIZED LINEAR INVERSION

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### ABSTRACT

The computation of detailed near-surface information using the refracted arrivals on seismic shot records has been the subject of much research since the early days of the seismic method. This near-surface information is used to compute initial static values for the seismic data.

In this paper, the redundancy found in CDP shooting is used to enhance the accuracy of the solution and reduce the effect of picking errors. An initial subsurface model is input by the user, normally consisting simply of a number of flat, constant-velocity layers. The model is then iteratively updated, by using a generalized linear inversion (GLI) algorithm, in such a way as to reduce the difference between the observed breaks and those calculated from the model. These model breaks are calculated by ray-tracing. Two advantages of the GLI algorithm over previous approaches are that the full redundancy of observed breaks reduces the sensitivity of the solution to picking errors, and that the final answer is constrained to be reasonably close to the input geological model.

The paper is divided into four parts. In the first part, we describe the procedure conceptually. In the second, we look more closely at the theory of the algorithm. In the third, model results are shown which demonstrate the convergence of the method through several iterations. Finally, the fourth part of the paper shows the performance of the method on several real data examples.

data itself are incapable of estimating these long-wavelength components accurately (Wiggins *et al.*, 1976), such residual statics programs may improve the stacking quality while leaving errors in structural times. An alternative procedure for estimating static shifts is to analyze the first breaks that result from refractions in the shallow low-velocity layers. If the shot-receiver offsets are properly distributed, those breaks should contain all the information necessary to correct both long- and short-wavelength static components.

The analysis of refracted arrivals has been the subject of research for many years, and a number of methods have been proposed. However, while these methods are most suitable for hand-interpretation of individual records, they tend to suffer from a number of limitations when applied to the computer-analysis of multi-fold data. These limitations include restrictive assumptions placed on the near-surface model, estimation of the long-wavelength static component only, difficulty in computer automation, sensitivity to errors in the picked arrival times, and inability to use the full redundancy implied in CDP shooting. In order to avoid these limitations, we have devised a method that develops a near-surface model by iterative ray-tracing.

### INTRODUCTION

The presence of shallow, low-velocity weathering anomalies can cause serious problems in the processing of seismic data. This paper is concerned with compensating for these effects by estimating a set of static corrections using refracted arrivals.

Two of the major effects that result from such near-surface conditions are the deterioration of the stack due to misalignment within the CDP, and distortion in the structural times of deep reflectors. Both these effects result from time delays introduced by geological layers of varying velocity and thickness. If this time delay pattern is Fourier-analyzed, it may be shown that components whose wavelength is shorter than the cable are primarily responsible for deteriorating the stacking quality, while longer-wavelength components cause structural errors in the deeper reflectors. Since it is known that residual statics programs that analyze the reflection

### ITERATIVE REFRACTION MODELLING

All refraction analysis methods assume some model of the near-surface geology. This is normally considered to be a series of layers whose thicknesses and velocities may vary both laterally along the line, and vertically with depth. The arrival times of the first breaks depend on the layer thicknesses and velocities, and the problem is to determine those parameters from the measured first arrivals. The direct approach is to perform some calculation using the observed breaks that yields the parameters themselves. An example of this is fitting a straight line through a selected segment of breaks, and measuring the slope and intercept (Knox, 1967). The problem with this type of approach is that it depends on a very restrictive model — for example, the line-fitting approach assumes that the refractors are locally flat with no short-wavelength anomalies.

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An alternative method is to iteratively build a model of the subsurface based on the information provided by the first breaks. The basic concept is simple and may be understood by referring to Figures 1 and 2. Suppose the actual earth consists of two low-velocity layers overlying some high-velocity material, as depicted on the left side of Figure 1. This would result in a set of breaks being recorded and plotted as "observed breaks" in Figure 2. Without knowing the actual earth configuration, we might guess that the layers are as depicted on the right side of Figure 1. By ray-tracing, we could then calculate the times at which the breaks are expected to occur, and these are plotted as "model breaks" in Figure 2. Generally, the model guess will be in error and there will be a discrepancy between the observed and model breaks. Figure 2 shows the "error" which is the result of subtracting the model break times from the observed break times. By analyzing this error, it is possible to estimate corrections to our original-guess model that will improve the correspondence between observed breaks and model breaks, ideally producing an error of zero.

The type of model used for our refractive modelling is shown in Figure 3. The near surface is composed of a number of low-velocity layers underlain by a high-velocity medium. The thickness of each layer can vary as a function of surface position. The velocity of each layer

can vary laterally, but not vertically. The velocity of the first layer is assumed as known. This is because the first layer is normally so thin that with typical shooting geometries there is not enough information to determine both the depth and velocity of this layer.

#### LINEAR INVERSE THEORY

The technique that we have described in the previous section is that of Generalized Linear Inversion. This is a powerful mathematical technique whose applicability to geophysical problems has been described in numerous papers (*e.g.*, Lines and Treitel, 1984). In the theory of the generalized linear inverse method (Backus and Gilbert, 1967), we pick a set of model parameters represented by the vector  $M$ , where:

$$M = (m_1, m_2, \dots, m_k)^T$$

These model parameters are assumed to yield a set of observations represented by the vector:

$$T = (t_1, t_2, \dots, t_n)^T$$

through some nonlinear functional relationship,  $A$ :

$$t_i = A_i(m_1, m_2, \dots, m_k), \quad i = 1, 2, \dots, n \quad (1)$$

In this particular problem, the vector  $M$  is the set of all the unknown thicknesses and velocities in the near-surface model, the vector  $T$  is the set of all observed first breaks, and the relationship  $A$  is the ray-tracing algorithm incorporating Snell's law. Starting from some

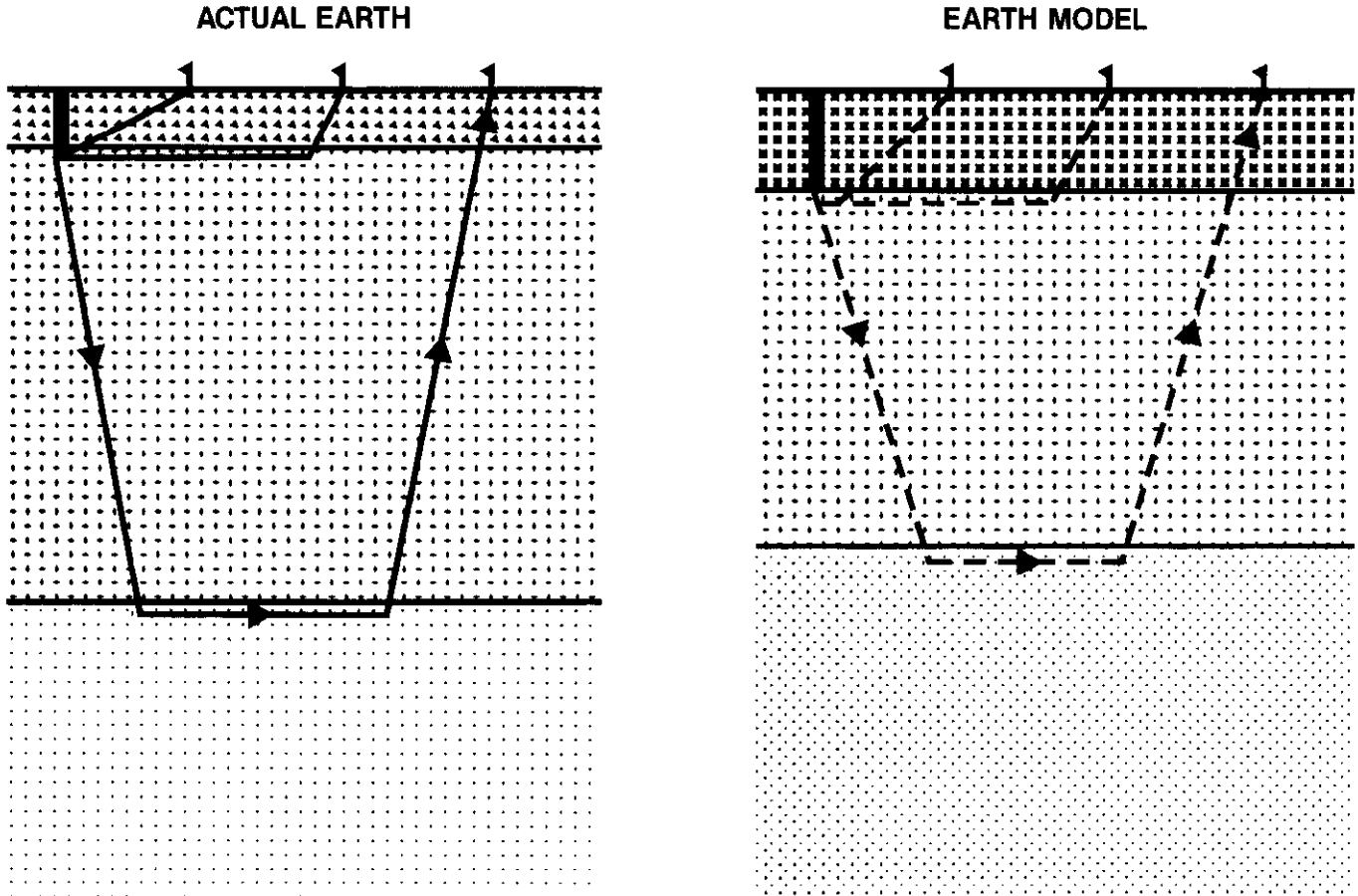


Fig. 1. Ray-tracing through actual earth model compared with ray-tracing through initial-guess model.

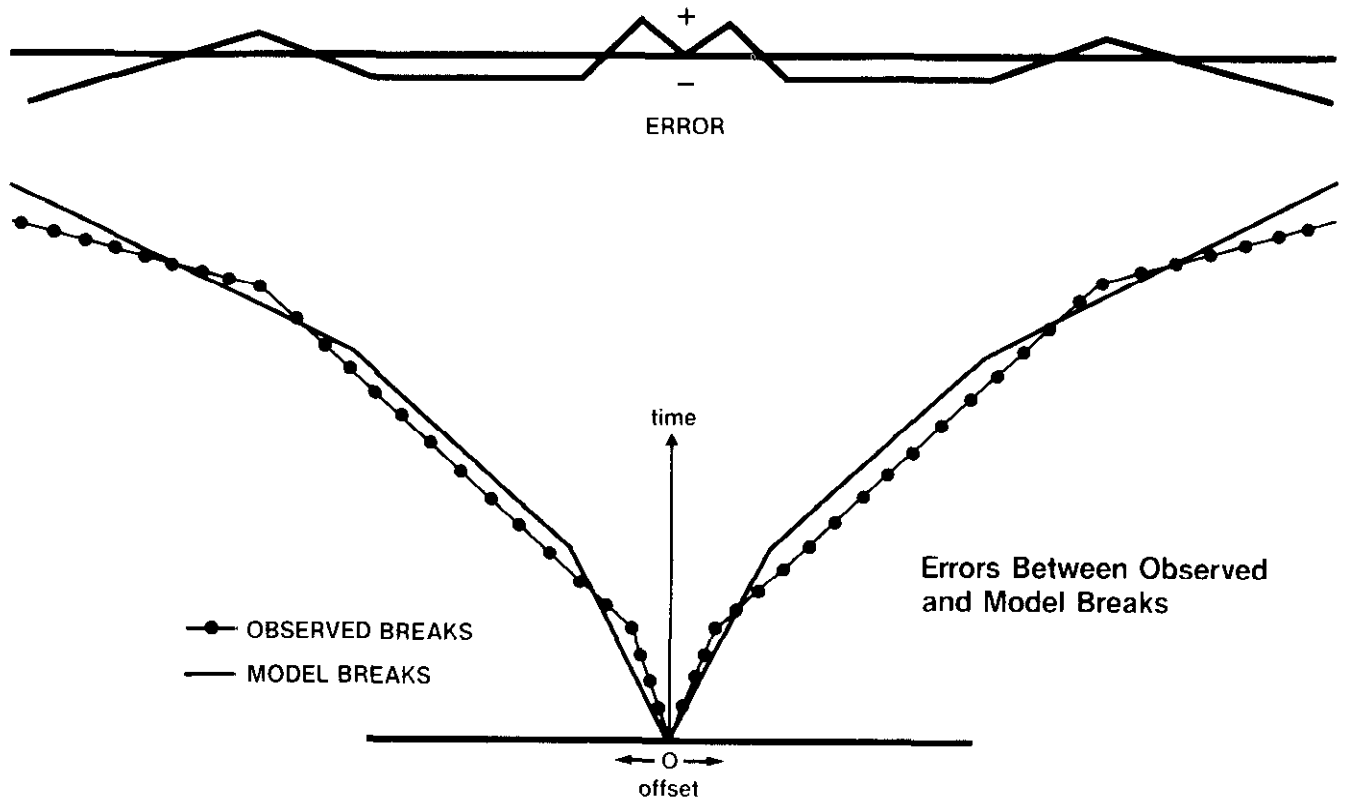


Fig. 2. Errors between observed and model breaks.

Refraction Modelling

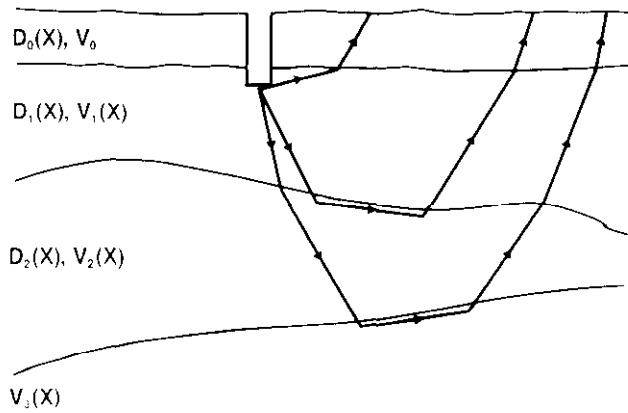


Fig. 3. Parameterization of model used for refraction analysis.

initial model guess,  $M^0$ , a set of arrival times is calculated and compared with the measured arrivals. As we have seen, Figures 1 and 2 show such a comparison, along with the residual error between the observed and calculated arrivals. The generalized linear inverse method reduces to analyzing this residual-error vector to determine a set of corrections to the model.

If we represent the error vector by  $\Delta T$  and the required model changes by  $M$ , then  $\Delta M$  is determined from the set of linear equations:

$$\Delta T = B \Delta M \tag{2}$$

where:  $\Delta T = A(M^{l+1}) - A(M^l)$

$B_{ij} = \partial t_i / \partial m_j$

$\Delta M = M^{l+1} - M^l$

$l = l$ th iteration

Note that in this equation, the matrix  $B$ , whose elements contain the derivatives  $\partial t_i / \partial m_j$ , is assumed to be known. Each element  $b_{ij}$  measures the sensitivity of the  $i$ th calculated first-break time,  $t_i$ , to changes in the  $j$ th model parameter  $m_j$ . In fact, those elements are functions of the model itself and must be recalculated on each iteration. The calculation is done analytically, assuming that the model velocities and depths vary slowly enough that the model may be considered locally linear.

As a result of multifold seismic recording, the linear system (2) is usually overdetermined, meaning that the number of first-break picks exceeds the number of unknown model parameters. This allows for some compensation for errors in the arrival-time picks. The solution is the least-squares fit:

$$M = (B^T B)^{-1} B^T T \tag{3}$$

which will ideally compensate picking errors whose probability distribution is a random, zero-mean Gaussian function. Unfortunately, picking errors are often large infrequent events due to cycle-skipping, and an algorithm must be implemented that detects unusually large error deviations after several iterations of the inversion sequence and deletes these picks.

In practice, the inversion of the very large matrix  $B^T B$  is neither practical nor necessary, since each application of equation (3) is one step in a series of iterations, and the exact solution  $M$  is not required in each step. For this reason, the solution (3) may be replaced by a suitable iterative approximation.

Figure 4 illustrates the general procedure. The user inputs an initial guess as to the near-surface configuration, which tells the program how many layers are expected and their approximate velocities and thicknesses. Generally this is a flat-layer model, although more complicated models may be required for rapidly varying situations. The program performs a series of iterations in which the model breaks are calculated by ray-tracing and compared with the measured breaks, then changes are made in the model and a new set of breaks calculated. This procedure is repeated until some acceptable correspondence is reached between observed and model breaks.

Refraction Analysis  
by Generalized  
Linear Inversion

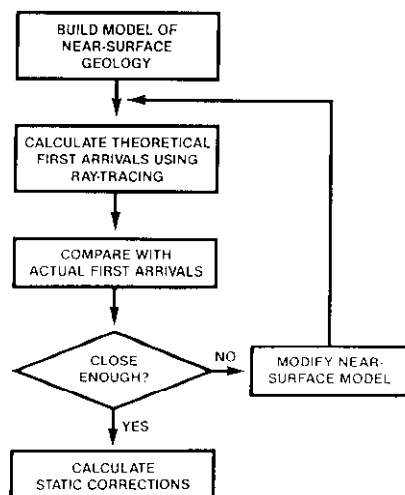


Fig. 4. General iterative modelling algorithm.

Although both velocities and depths may vary with each station, as implied earlier in Figure 3, the algorithm can be made more robust by forcing a more slowly varying model solution. This increases the number of observed breaks used to update any given model parameter and diminishes the effect of random picking errors. While the layer velocities may be expected to vary slowly, the layer thicknesses should not and, in fact, their high frequency variations are precisely the cause of the short-wavelength statics mentioned earlier. Forcing a smooth model means that only the longer-wavelength components are accounted for with the iterative modelling technique. The short-wavelength components can be estimated at the end of the process as simple surface-consistent time delays in the final residual error (Chun and Jacewitz, 1981).

#### SYNTHETIC EXAMPLE

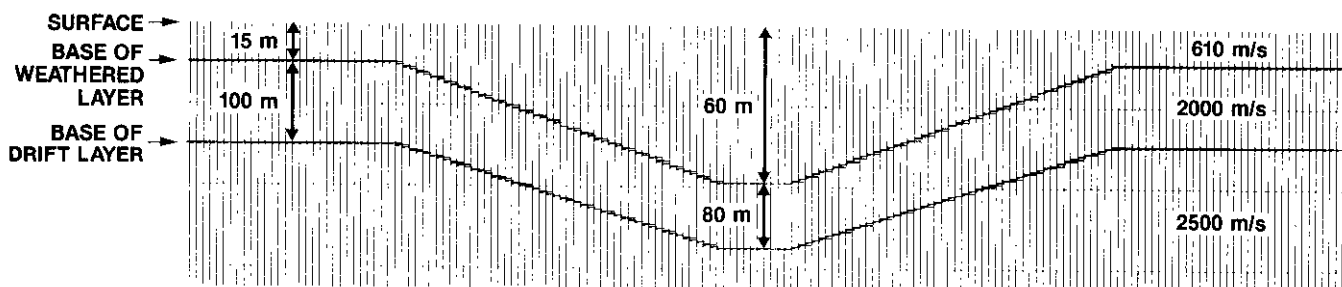
In Figures 5 to 7, we illustrate the method on a syn-

thetic data example. Figure 5 shows a hypothetical near-surface model, consisting of two low-velocity layers overlying a higher-velocity medium. A large weathering anomaly is simulated at the centre of the model. By ray-tracing through this model, we have created a set of refracted arrivals. These arrivals will be analyzed by the inversion algorithm in an attempt to recover the original model.

Figures 6 and 7 show the model and residual errors at the start and end of the calculation. At the top of Figure 6, the original synthetic is shown, followed by the initial guess, which was two layers of uniform thickness and velocity. The velocities of both the second low-velocity layer and the high-velocity medium are in error, as are the layer thicknesses. The bottom part of Figure 6 shows the residual error that results from subtracting the model breaks (calculated by ray-tracing through the initial-guess model) from the original "measured" breaks. The shooting geometry was simulated as a 96-trace cable, so that each of the patterns in the lower part of Figure 6 consists of 96 points showing the error in each trace in exactly the same manner as at the top of Figure 3. If the initial guess had been correct, the residual errors would all be zero, and the patterns would be flat lines along the timing lines in the figure. Because of the errors in the initial guess, the patterns are a series of broken lines corresponding to the different refracting paths. Roughly speaking, a nonzero slope on a line segment indicates a velocity error while a time shift indicates a thickness error.

After five iterations, in Figure 7, the residual error is negligible, and the resulting model looks very close to the input. In this simulation, the inversion algorithm was constrained by using a six-station smoother — meaning that as each parameter was updated, the measured breaks corresponding to three stations on either side were used in the calculation. As a result, the final answer does not have the sharp edges shown on the original synthetic. Although that has caused very little residual error in this synthetic example, the smoothing on real data sets will usually leave short-wavelength surface-consistent static patterns in the residual error plots, which can then be removed as simple time-shifts and used to correct the seismic data further.

The synthetic example shows that the inversion algorithm, under ideal conditions, can invert the ray-tracing process and recover the original model. However, for the method to work on real data, three conditions must be met. First, the model must be sufficiently complex to describe real behaviour adequately, but must still have significantly fewer parameters than observations (real breaks). This means that the user must specify the correct number of layers, and that the geology must vary slowly enough that ray-tracing is appropriate. A model with a very large number of layers would force the program to estimate too many parameters for the number of observations, and a mathematical instability could result. Second, the initial guess must be close



### Synthetic Near-Surface Model

Fig. 5. Model used to generate synthetic refractions.

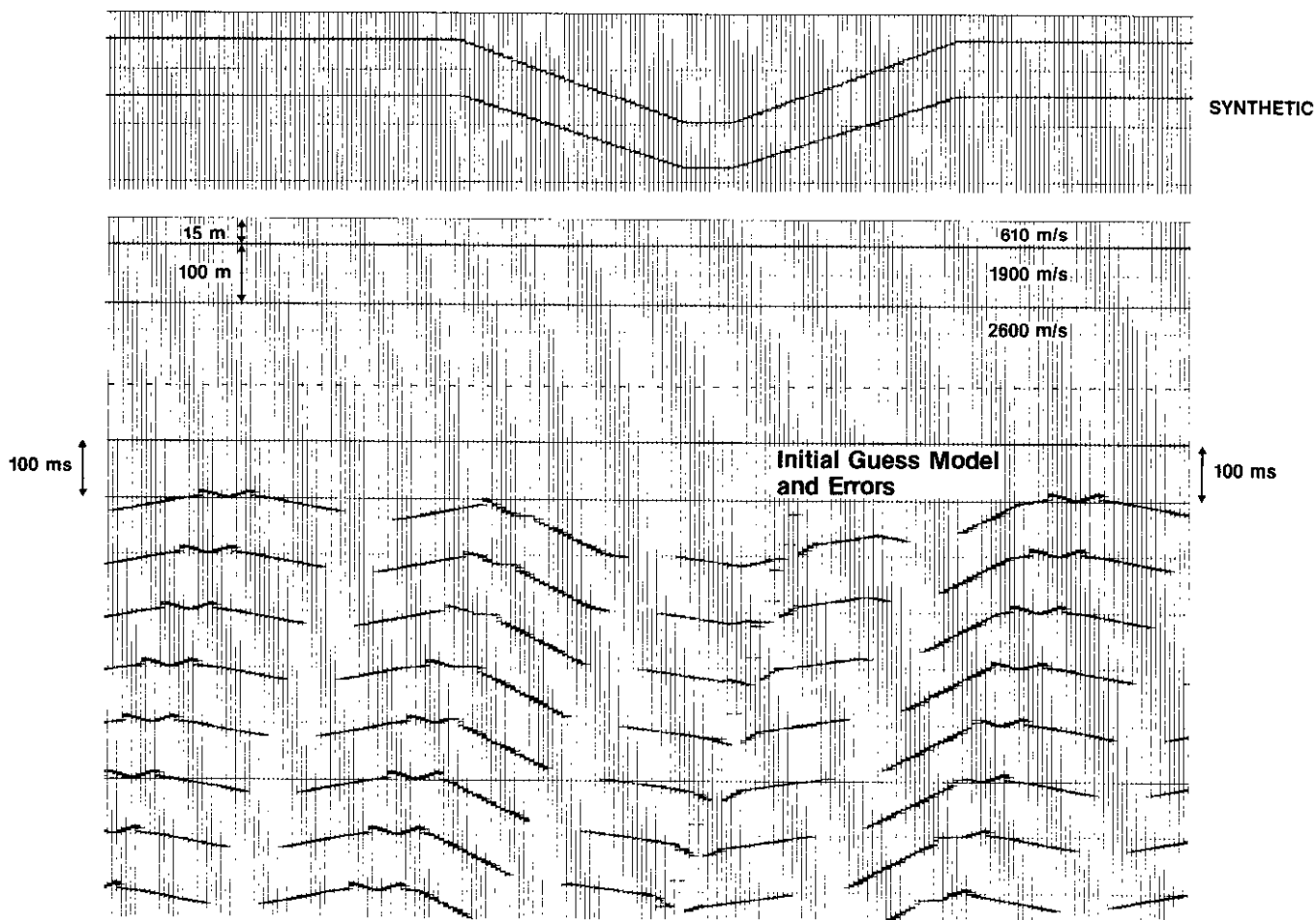


Fig. 6. Initial-guess model and residual errors before iterative modelling.

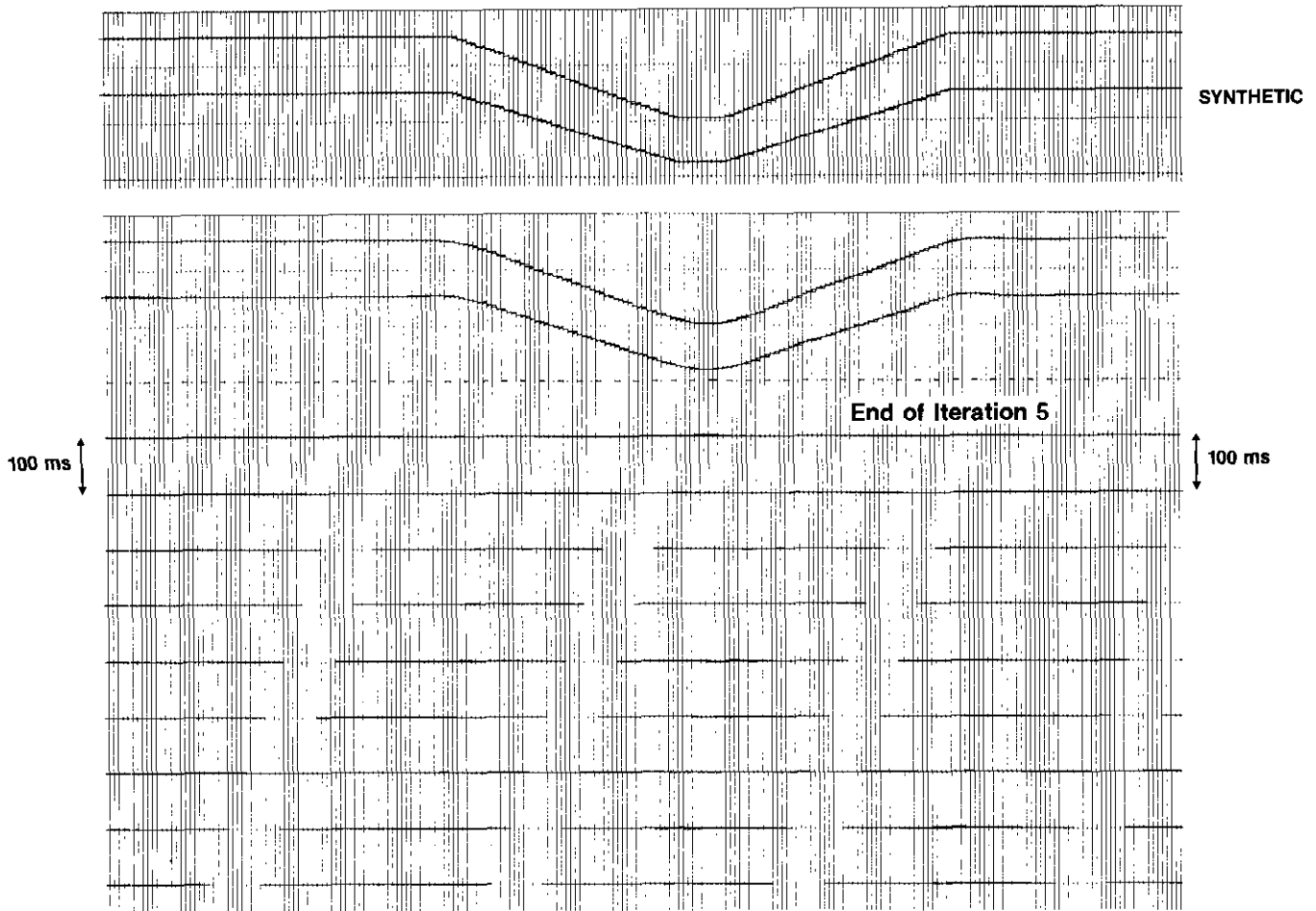


Fig. 7. Final result after five iterations.

enough to the correct answer to allow convergence within a reasonable number of iterations. (Fortunately, in most real data situations, the near-surface geology is well enough known that neither the first nor the second condition is a problem.) Third, the input first-break picks must be free of gross errors. The inversion algorithm is ideally suited to handle small random errors, but picking errors are often large cycle skips. If the frequency of cycle skips is not too large, the program can be used to detect picking errors by performing several iterations and searching for excessively large residual errors. This procedure has in fact been used with good success.

In general, it can be stated that the method is ideally suited to those areas where the near surface is restricted to two or three layers whose parameters vary over a predictable and limited range. Experience has shown that this describes a large percentage of land seismic

data, and that the algorithm is a robust and reliable tool requiring very little human intervention.

#### REAL DATA RESULTS

The interpretation algorithm will be demonstrated on two real data sets. The first was chosen because it contained a particularly large weathering anomaly, as can be seen in Figure 8. The breaks themselves were very clean and the automatic picking algorithm had little trouble. At the top of Figure 9, the initial-guess model is displayed in the form of two-way travel times to the base of each of the two low-velocity layers. The initial guess consisted of uniform layers of constant velocity. The residual errors from three shots in the vicinity of the weathering anomaly are shown at the bottom of Figure 9, where the surface consistent pattern is evident.

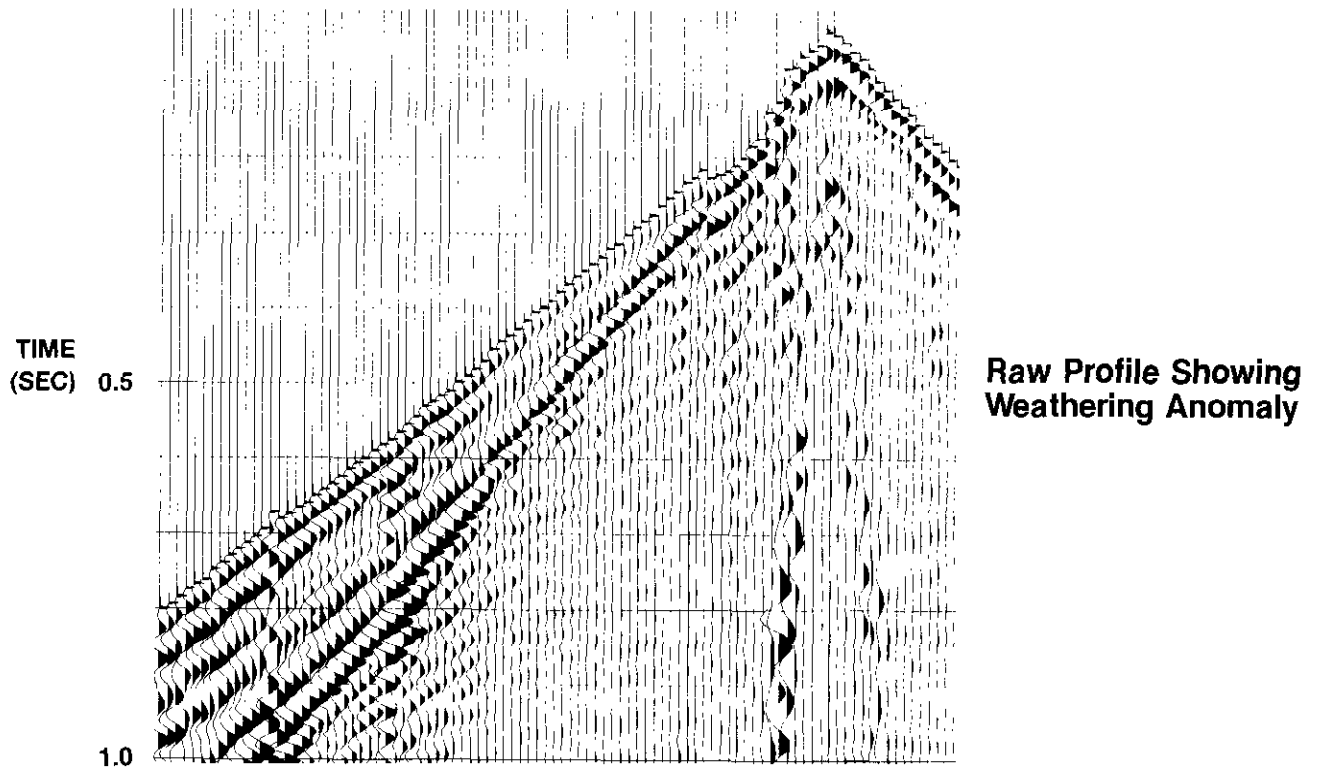


Fig. 8. Single profile from real data set with weathering anomaly.

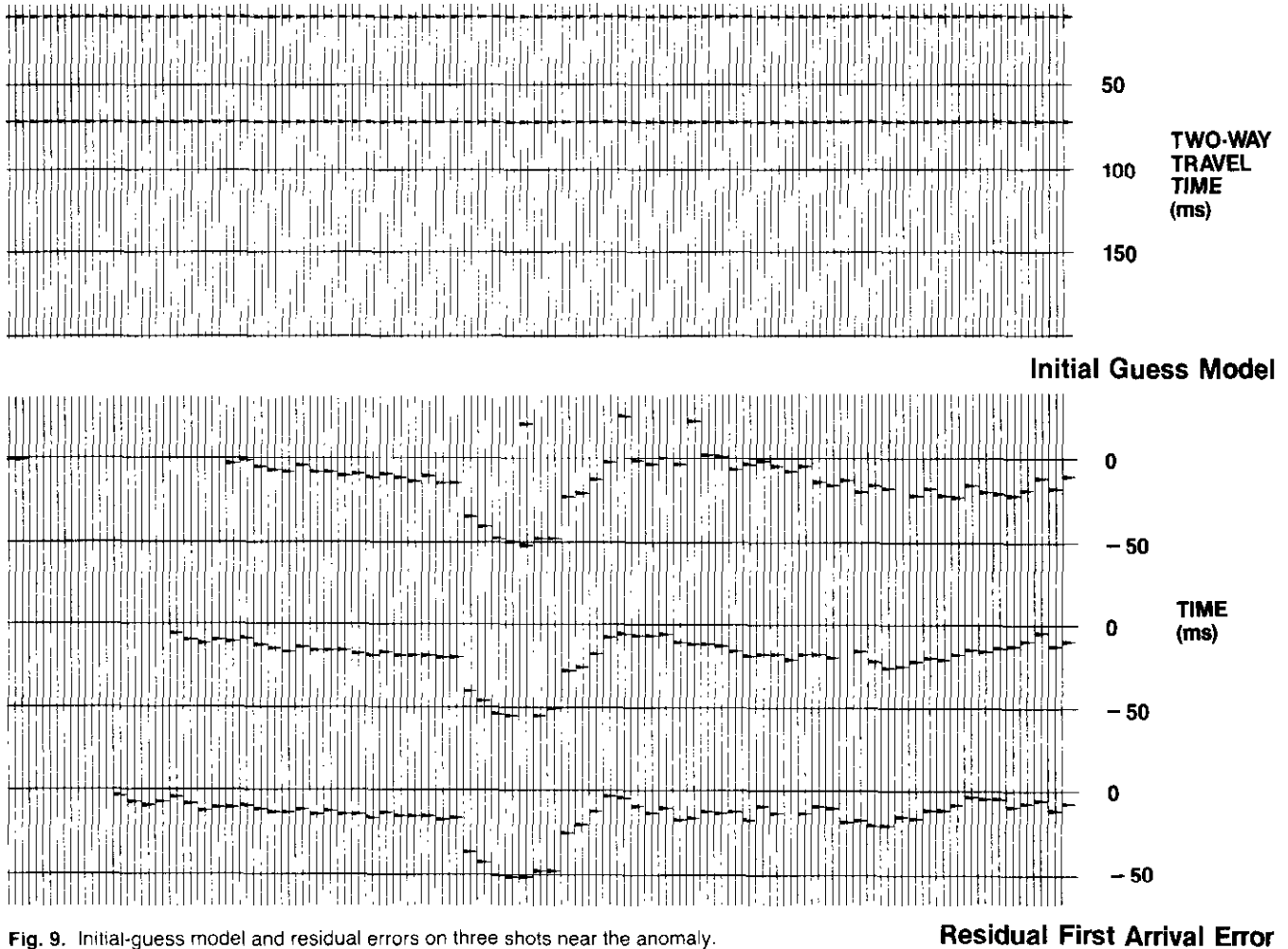


Fig. 9. Initial-guess model and residual errors on three shots near the anomaly.

After five iterations, the algorithm has converged to the result shown in Figure 10. The thicknesses of both layers have changed considerably from the initial guess, including, of course, the large anomaly. Because a six-station smoother was used in the depth calculation, the sharp step on the left side of the anomaly was modelled as a smooth drop. Consequently, a short-wavelength residual correction remained in the error pattern after ray-tracing through the smooth model. This pattern was estimated as a surface-consistent time delay and is plotted in the central panel of Figure 10. In correcting the seismic data, the short-wave statics will be added to those calculated by using the final near-surface model. At the bottom of Figure 10, we see that the residual

errors are very close to zero, indicating a good fit between the model breaks and the real breaks.

Figures 11 and 12 show a comparison between the brute stacks generated before and after application of the refraction statics. The major differences between the two can be seen just under the muted zone, where the continuity of the reflector at 1.0 s has been improved by the application of some very large statics. Although a residual statics program will further improve either of the sections, it is clear that removing the large weathering patterns in the early processing will be helpful both in the velocity analysis stage and in minimizing the risk of cycle skipping in further statics calculations.

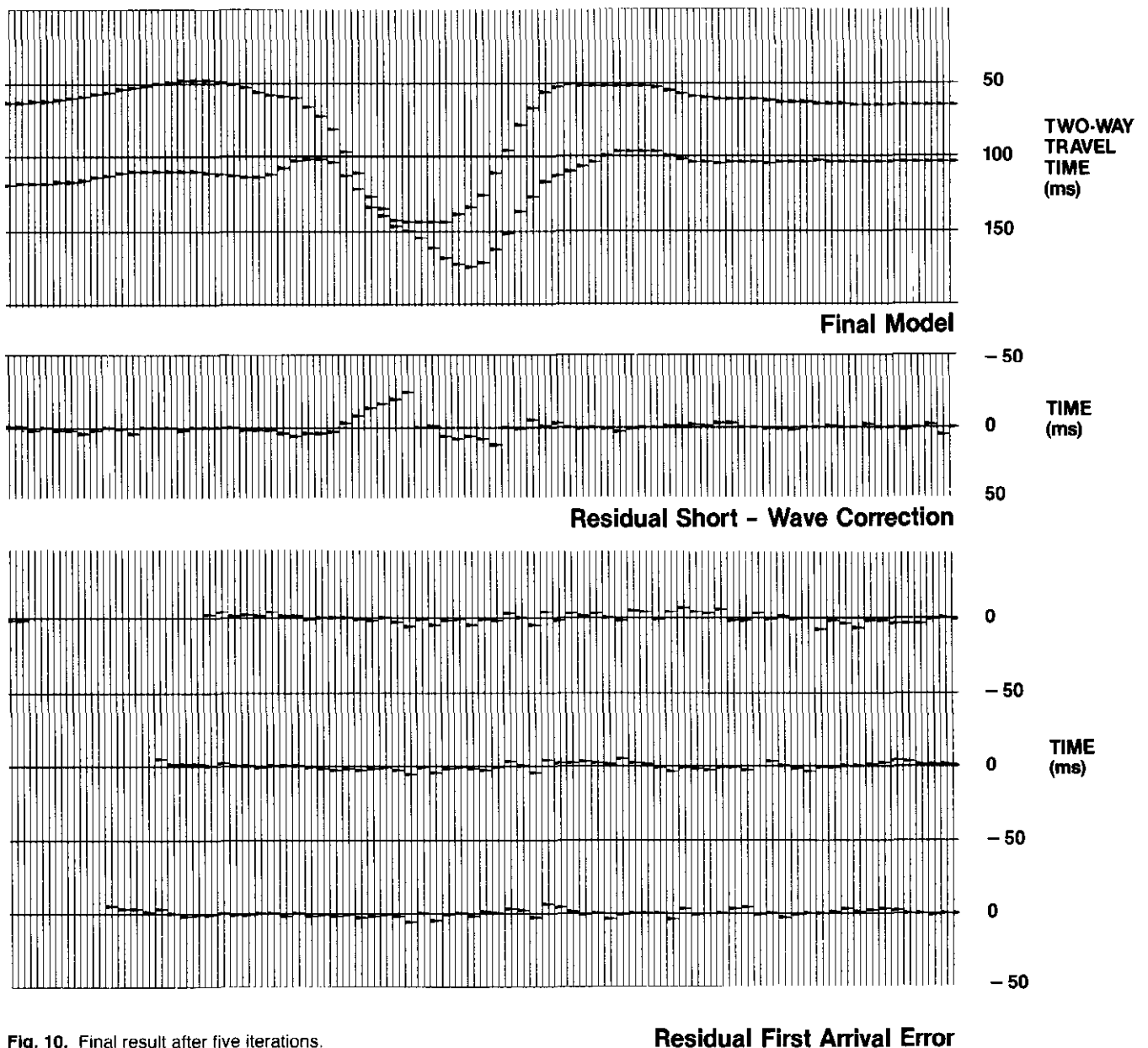


Fig. 10. Final result after five iterations.

Residual First Arrival Error

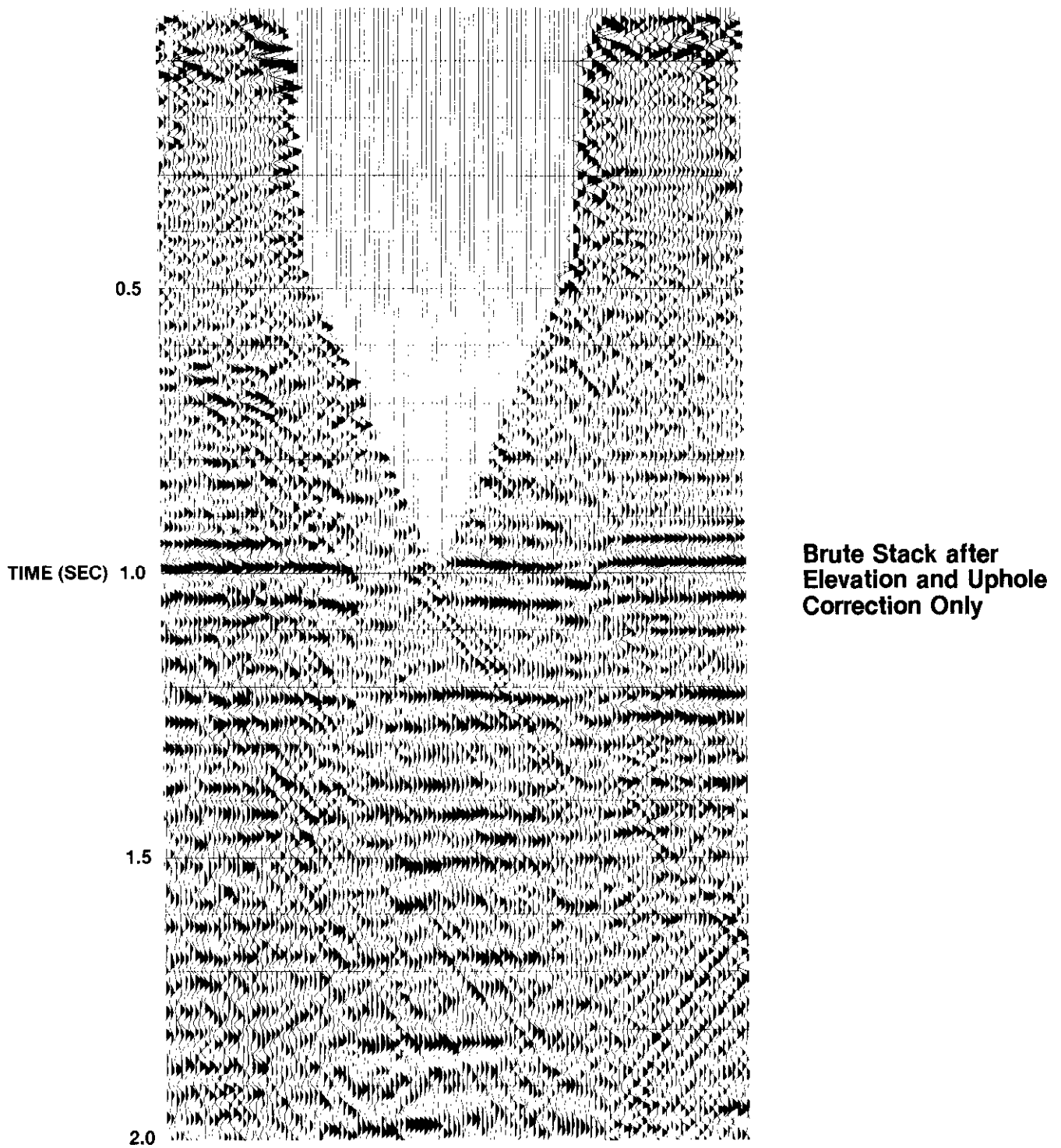


Fig. 11. Brute stack after elevation and uphole correction only.

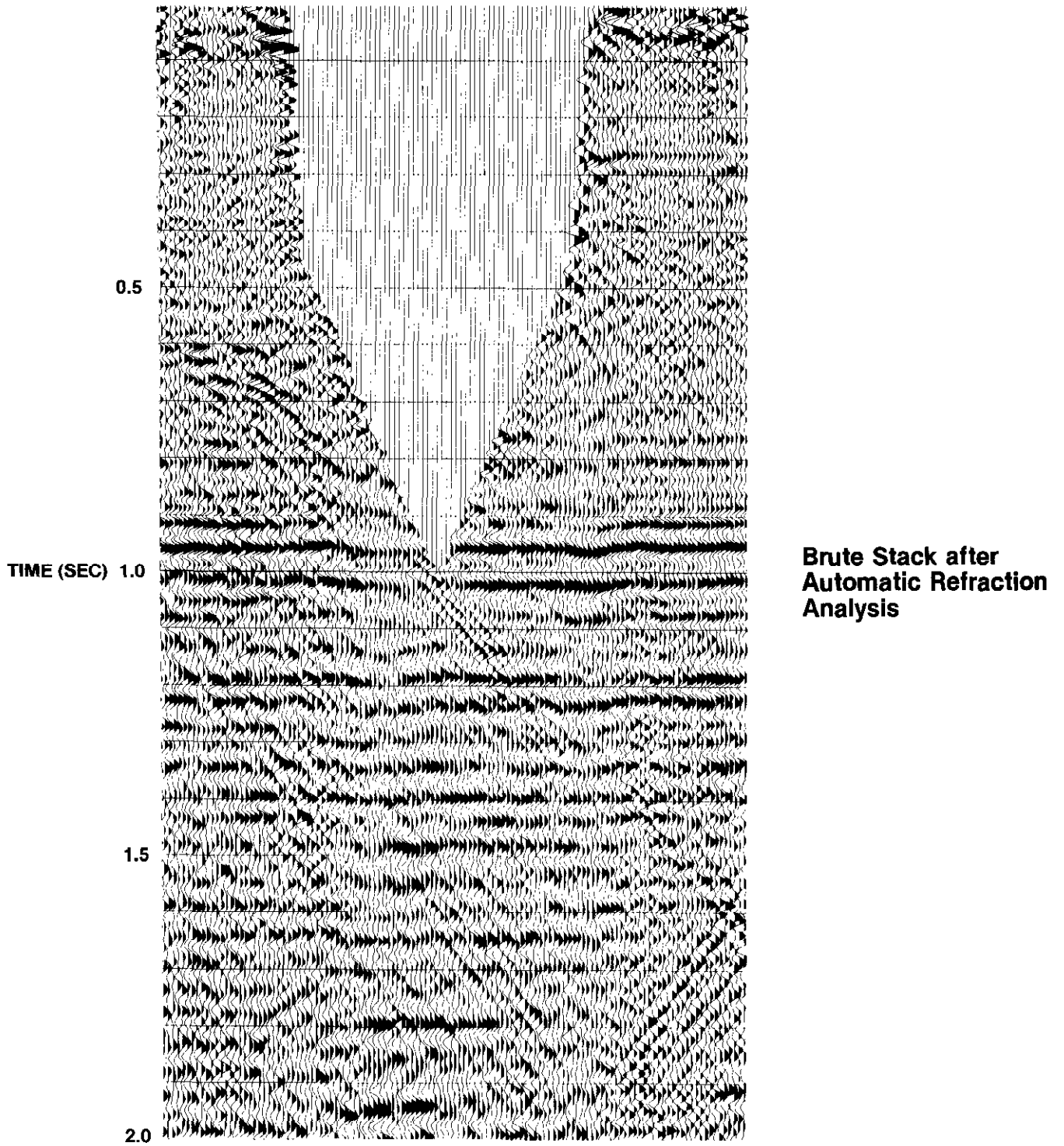


Fig. 12. Brute stack after automatic refraction analysis.

The second real data example was chosen because it displayed both long-wavelength and short-wavelength static anomalies. This is demonstrated quite clearly in Figure 13, which is a brute stack of the line with elevation and uphole corrections applied. The long-wavelength static appears as a slowly varying structural component, with a maximum variation across the line of about 20 ms. The short-wavelength component results in loss of continuity on some of the weaker reflectors, especially the data above 900 ms, and also in a vertical swath to the left side of the low-coverage gap.

In Figure 14, we see the same stack after automatic refraction analysis. In this result, both the long- and the short-wavelength statics solutions have been improved. The removal of the long-period static results in a more structurally valid section without the need to resort to the artificial solution of flattening on a particular event. It should be made clear that residual statics programs based on correlation of reflection zones are theoretically unable to solve for this long-period static pattern.

The short-period statics solution results in improved data continuity, both in the events at 900 ms and above, and also in the vertical swath of data in the central part of the line. For a closer look at the results, Figure 15 shows a slice of the data before and after refraction analysis, and also the near-surface model that was produced. Notice that the structure of the near-surface model is reflected in the long-period static pattern that was removed. Also, the vertical stripes of poor reflection continuity seen in the original data correspond to severe, short-wavelength weathering pockets.

Although the automatic refraction analysis has done a reasonably good job in finding the short-period statics, it does not replace the more traditional residual statics methods entirely. Rather, it should be used in conjunction with these other methods for optimum results. This is shown in Figure 16, where the stacked section for Figure 14 has been displayed after a residual statics analysis. Notice that this final process has fine-tuned the statics solution of the previous stack. It should be re-emphasized, however, that this residual statics program would not have solved the long-period static seen in Figure 13.

## CONCLUSION

In this paper we have proposed a method for the computation of static corrections based on a generalized linear inversion technique that iteratively models the near-surface by using refracted arrivals. The major advantages of this approach over traditional methods are:

- By using the full CDP redundancy, the solutions are stabilized and the effect of picking errors reduced.
- Previously known information about the near-surface geology, as well as any geological constraints, can be readily incorporated.
- Restrictive modelling assumptions are minimized.
- The algorithm can be readily automated and requires little user intervention.

Two real-data cases were shown that verified that both the long- and short-wavelength static components were instrumental in improving the stacked section. Although a residual statics pass is still necessary to fine-tune the seismic image, removing the major static anomalies in the early processing stages corrects the structural timing of events and ensures optimum processing in the later stages.

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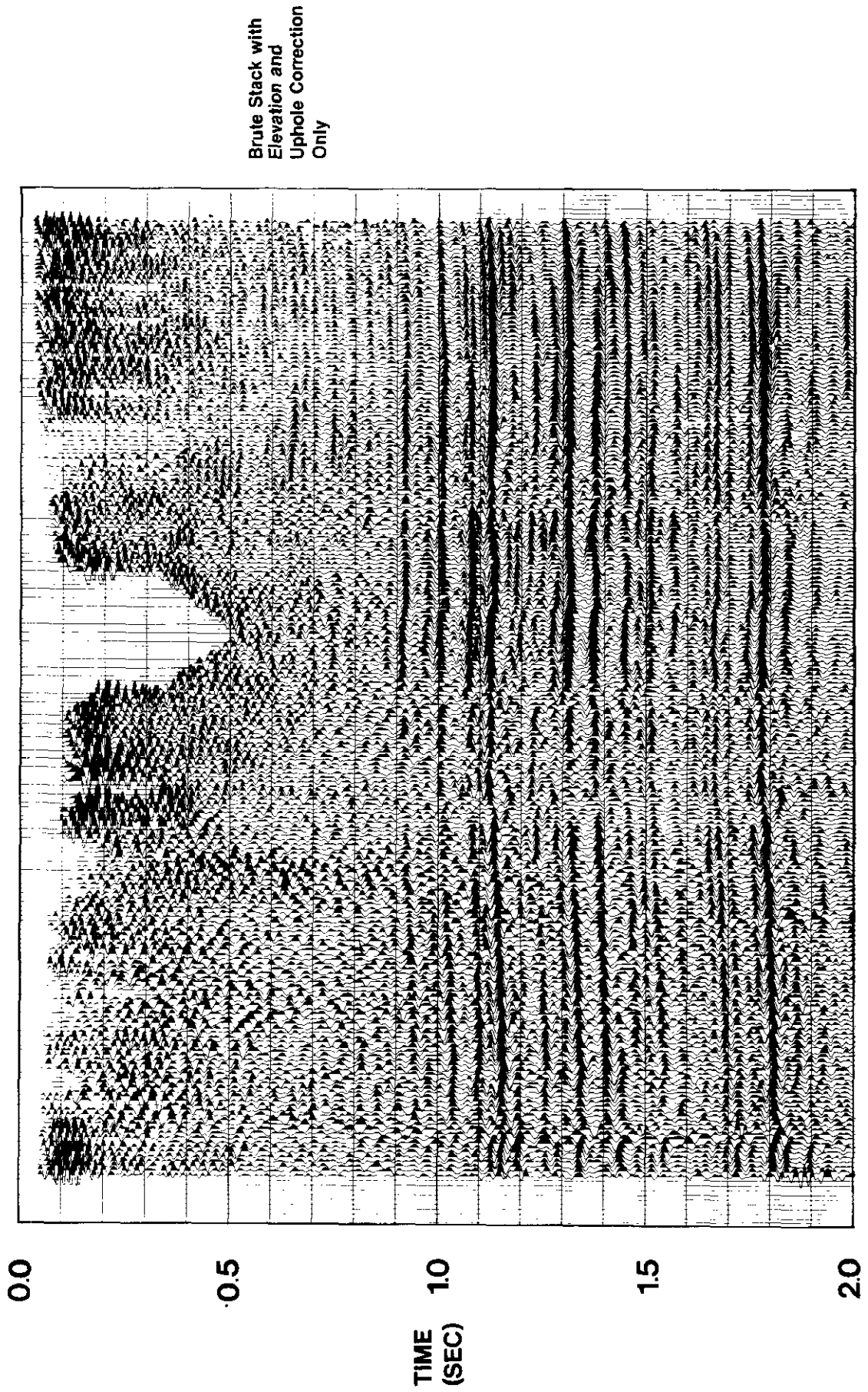


Fig. 13. Second example: brute stack with elevation and uphole correction only.

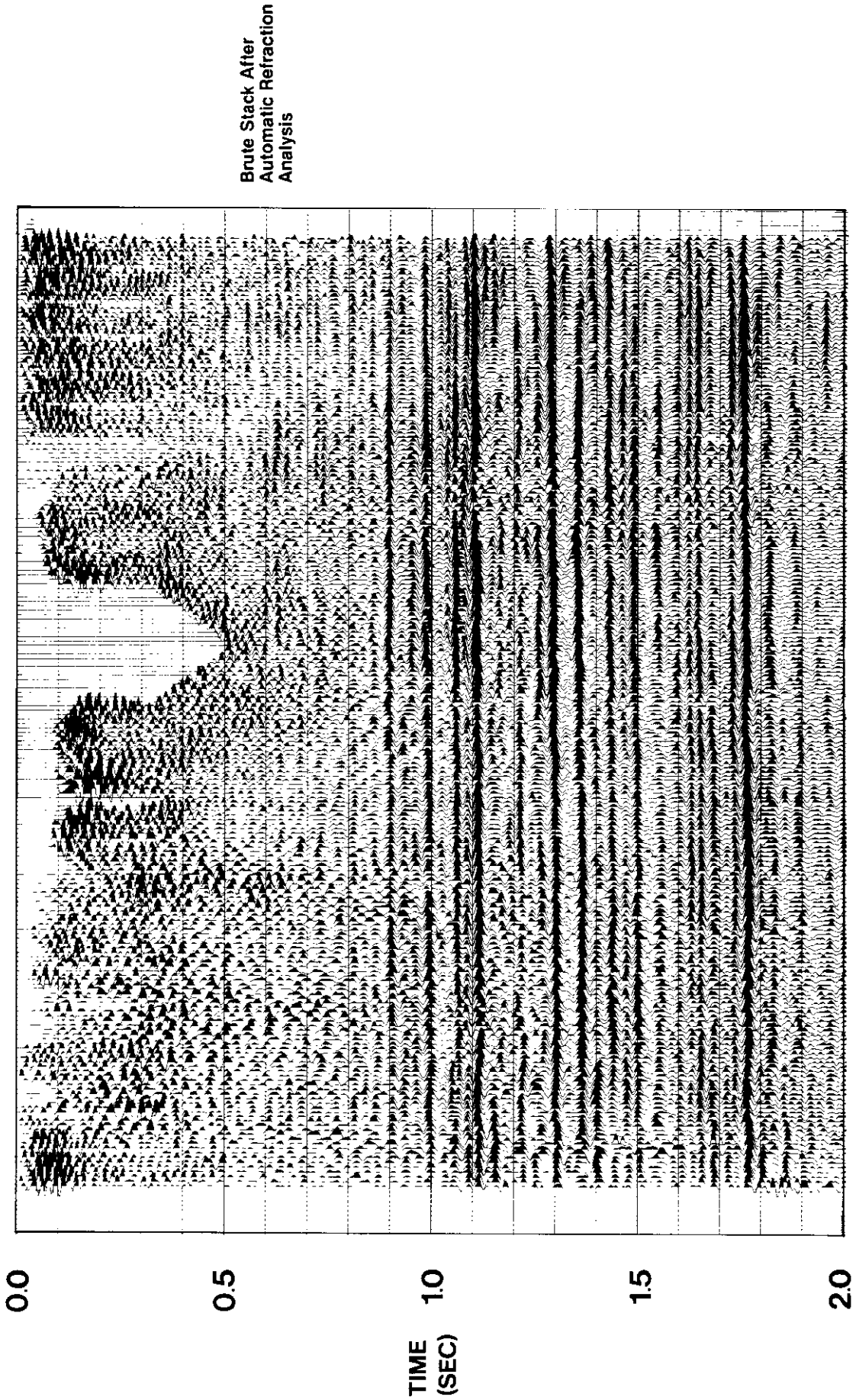


Fig. 14. Second example: brute stack after automatic refraction analysis.

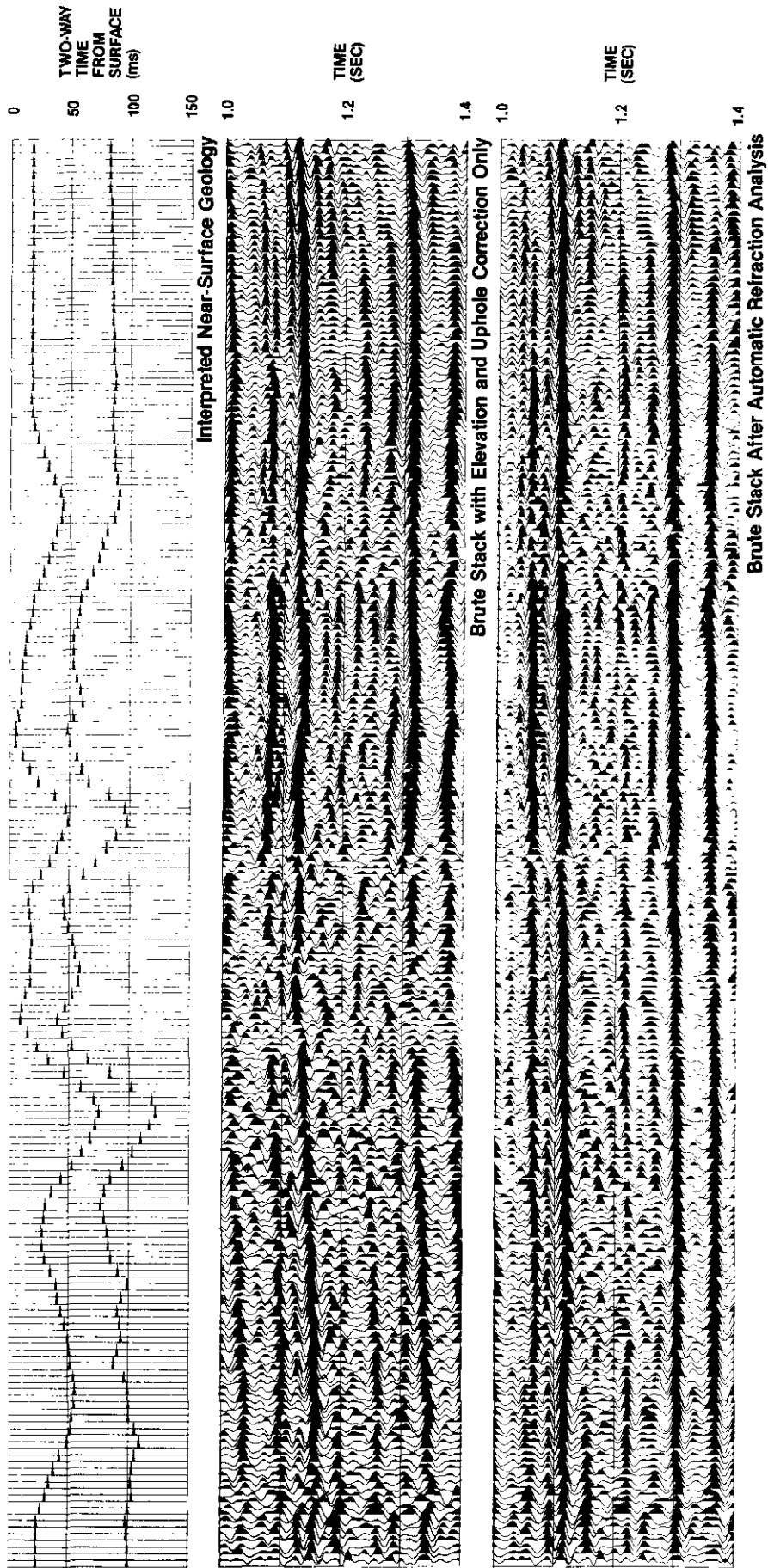


Fig. 15. Interpreted near-surface geology with brute stacks before and after refraction analysis.

**Final Stack After  
Residual Statics  
Analysis**

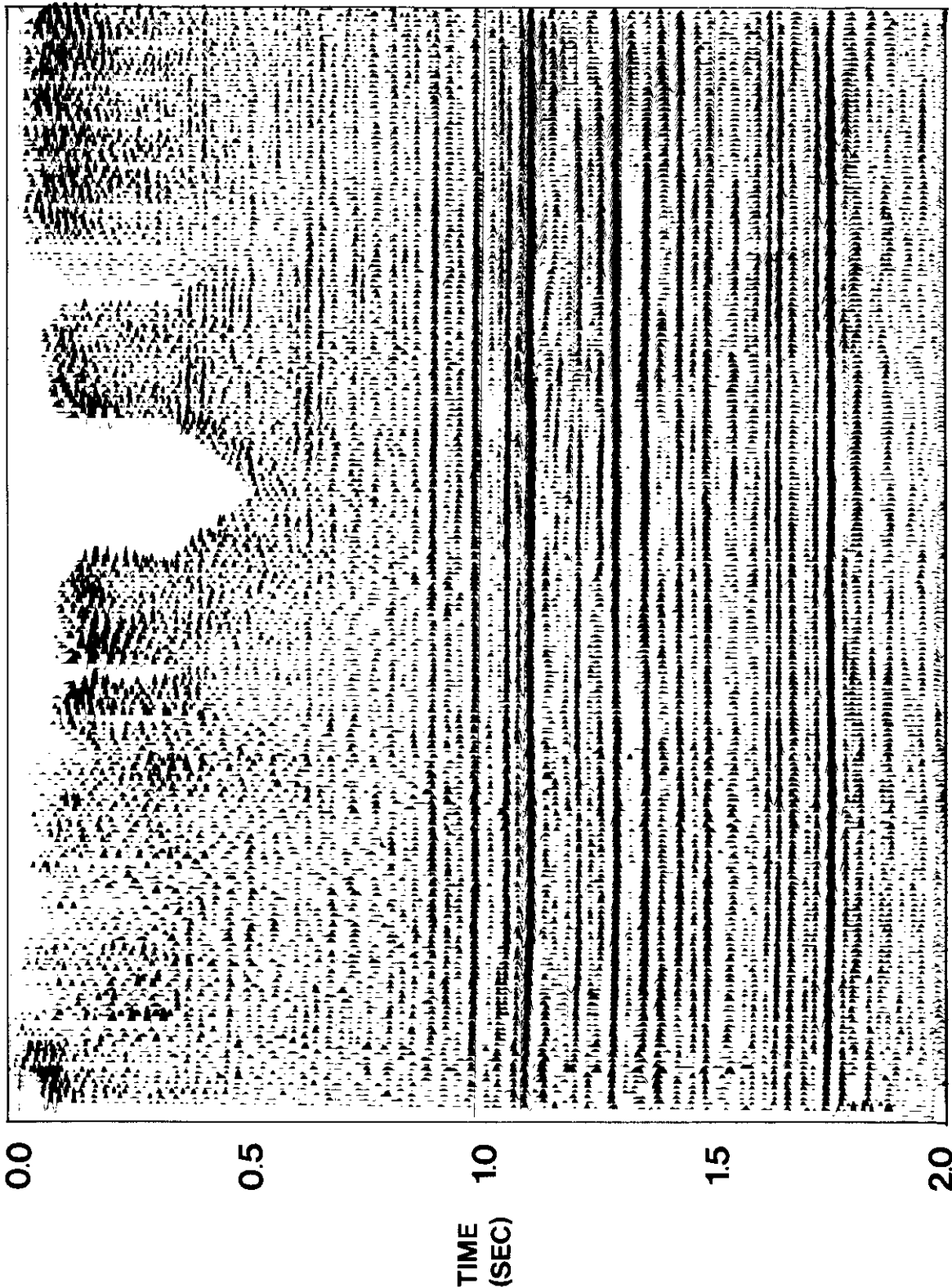


Fig. 16. Final stack after residual statics analysis.