A SIMPLE, EFFICIENT METHOD FOR THE CALCULATION OF TRAVELTIMES AND RAY PATHS IN LATERALLY INHOMOGENEOUS MEDIA

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Abstract

An efficient method for the determination of ray paths and traveltimes in laterally inhomogeneous media has been developed. Models use plane interfaces with any dip to form arbitrary two-dimensional layers or blocks. The specification of a linear velocity gradient within each block enables rays to be traced from one boundary to the next in one step, thereby eliminating the multiple calculations usually required within one layer. Rays incident on a boundary are either reflected or refracted according to Snell's law. Critically refracted rays are incorporated into the formulation by noting the particular ray which impinges at any interface within a specified (small) angle of critical incidence. Beyond this point of intersection, critically refracted rays are generated at regular intervals along the boundary to simulate the upward travelling energy associated with true head wave arrivals. Converted phases, multiple reflected ray paths, and sub-critical incidence reflected arrivals are not considered.

The method is illustrated with two examples. The first includes a laterally homogeneous crustal model with significant velocity gradients and a low velocity zone. For the specified velocity-depth distribution, traveltimes determined by the ray tracing algorithm agree well with those calculated directly by a standard approach. The second model represents a structurally complex crustal cross-section which includes dipping interfaces, vertical faults and individual blocks of material with different velocities. Traveltimes derived by ray tracing through the model agree closely with those picked from the observed record section. The simple, efficient algorithm allows many such models to be proposed, and tested, thereby facilitating the interpretation of data from geologically complex regions.

Introduction

The ultimate objective of most seismic surveys is to provide a geological model of the subsurface structure. In so doing, the traveltime and sometimes the amplitude content of the seismic data are used to obtain the model. Some procedures, including the classical approach to interpretation of refraction seismograms (Dobrin 1976, p. 296), directly invert the traveltime data to provide a velocity versus depth curve for the region. More sophisticated mathematical methods, including extremal inversion and generalized linear inversion, have been implemented in recent years. Kennett (1976) provides a summary. To incorporate both the kinematic and dynamic aspects of the seismic data into the interpretation, synthetic seismograms and a trial-and-error procedure are widely used. Many synthetic seismogram computer routines exist and these vary in their complexity and method of calculation (for a representative selection, see Hron and Kanasewich 1971; Fuchs and Muller 1971; Wiggins and Helmberger 1974; Wiggins 1976; Chapman

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1978). However, all of these procedures assume lateral homogeneity in the subsurface model, or, in the case of classical refraction interpretation, plane dipping layers of constant velocity.

Often geological structures cannot be represented reasonably by a laterally homogeneous earth model. In such cases, seismic ray paths are traced through a proposed geological model, the traveltimes and distances are determined and a comparison with the observations is made. Many of these approaches consider only the simulation of traveltimes (Jacob 1970; Sorrells et al. 1971; Scott 1973; Will 1976; and Gebrande 1976) although invariably different computational techniques are employed. Other, more sophisticated approaches, such as those described by Yacoub et al. (1970), Clee et al. (1974) and Green (1976), include estimates of the seismic wave amplitude as well as the traveltime information. Methods for the direct calculation of synthetic seismograms for models with laterally and vertically varying elastic parameters are being developed (Odegard and Ambos 1978; Marks and Hron 1978). Use of these techniques should be of considerable aid for interpreting complex seismic record sections.

Some recent land and marine seismic profiles recorded in British Columbia and off the west coast of Canada showed considerable evidence for lateral variations in the subsurface structure. We obviously required an interpretation method to deal with such complex geological areas. Since we intended to use a trial-and-error procedure in which traveltime-distance information was calculated for many models, it was important that the technique used be simple and efficient for implementation on a computer. Many of the existing calculation techniques mentioned above appeared to be overly complex. However, the methods described by Gebrande (1976) and Will (1976) seemed somewhat simpler and had the advantage of being used for interpretation of data similar to ours (see Grubbe 1976). Our approach resembles most closely that described by Gebrande (1976). Seismic ray paths and traveltimes for a laterally inhomogeneous medium are determined through an efficient, easy-to-use computer algorithm. Interest shown in our technique and the results achieved with it have prompted us to write this paper.

Method Of Calculation

Our approach uses plane interfaces with any dip to form arbitrary two-dimensional layers or blocks. Linear velocity gradients are required but arbitrarily small gradients may be used to simulate constant velocity blocks. The routine traces a ray from one boundary to the next in one step, eliminating the multiple calculations of the iterative methods (Gebrande 1976; Will 1976). The traveltime and horizontal distance increments for each ray path segment between two boundaries are individually summed to yield values for a traveltime versus distance plot which shows all surface arrivals, including critically refracted or pseudo-head waves. The model and the ray paths traced through it form a second plot. Converted phases, multiply reflected ray paths and sub-critical incidence reflection arrivals are not considered. The validity of a given model with respect to a set of observed data is judged by the fit of the first and subsequent arrivals on the T-X plot. A qualitative measure of amplitudes may be obtained by noting the number of rays per unit distance arriving at the surface. Furthermore, the relative lengths of the wave trains after onset may be estimated using the number of secondary arrivals and their traveltimes at the different distances.

Every boundary in the model is given a constant velocity along its length and a non-zero, linear velocity gradient normal to its direction of dip. As a result, all ray paths within a layer are circular arcs. The equation of the circle depends on three parameters: 1) the velocity, VOUT, at the boundary from which the ray departs, 2) the prevailing gradient, GRAD, and 3) the departure angle, IOUT, measured with respect to the gradient. The radius of the circle, RHO, is given by (Telford et al. 1976, p. 274):

$$\mathrm{RHO} = \mathrm{VOUT}/(\mathrm{GRAD}^*\mathrm{Sin}(\mathrm{IOUT})).$$

Once a circular path is defined all layer boundaries are checked to see which ones, if any, intersect the arc. Since no rays are allowed to travel from right to left, the horizontal (X) coordinate of a ray must increase monotonically. The boundary-ray intersection point which has the smallest horizontal distance value (greater than that of the departure point) is selected as the end of the current circular arc. Snell's Law is applied using the velocities on either side of the new boundary to find the new angle of departure. This angle, along with the new velocity and gradient determine another circular path and the process continues.

Velocities are calculated using (from Telford et al. 1976, p. 273)

$$V = V(0) + GRAD*Cos(I)*Z$$

where V(0) is the velocity along the nearest overlying boundary, GRAD is its gradient, I is its dip and Z is the vertical distance to the boundary from the point in question. This formula gives the incident or exiting velocity for descending or ascending rays, respectively. The velocity assigned to the boundary itself is usually taken as the other velocity required by Snell's Law.

Models with lateral inhomogeneities can have an interface along which there exists no constant velocity. For example, the near-vertical boundary between two abutting horizontal layers has a velocity that increases with depth because of the non-zero gradients on either side. Such boundaries are assigned a velocity of zero. When a zero-velocity boundary is intersected the program probes upward to find the constant-velocity segments which govern the properties within the blocks on either side of the intersection point.

The traveltime for an individual arc segment depends on: 1) the gradient, 2) the initial ray departure angle measured with respect to the gradient, and 3) the ray intersection angle on the next boundary measured with respect to the same gradient. If the above three parameters are symbolized by GRAD, IOUT and IIN respectively, then (from Gebrande 1976) the traveltime, DT, is:

$$DT = (1/GRAD)* \{Arctanh\ [Cos(IOUT)] - Arctanh\ [Cos\ (IIN)]\}.$$

Thus, the program constructs ray paths out of long arc segments. This technique is in contrast to some other ray-tracing algorithms which use very small distance increments while travelling through a detailed grid of velocity-depth points.

Critically refracted or head waves, propagating along layer interfaces, send energy back up to the surface in defiance of conventional ray theory. Spherical wavefronts are required to account for this phenomenon. Nevertheless, arrivals associated with head waves must be incorporated into the calculations because they are usually first arrivals on T-X plots. For a series of rays emanating from a source point over a range of discrete take-off angles, probability and computer round-off error make the possibility of one ray impinging at the exact critical angle of refraction very unlikely. An approximation is made. Initially, rays leave the source at shallow departure angles, measured with respect to the horizontal surface, and travel away in a nearly horizontal direction. As the departure angle becomes larger, the rays will follow a steeper path into the earth. Thus, on most layer boundaries, there will be a place where one ray reflects from, and the next ray refracts through, the interface. A critically refracted ray must lie somewhere between the two. If the first refracted ray intersects the boundary at an angle within some specified (small) angle of the theoretical critical angle, pseudo-head waves are generated. The first such critically refracted ray emerges from the boundary a short distance from this intersection point. Subsequent critically refracted rays are generated at regular intervals along the boundary to simulate the upward travelling energy associated with true head wave arrivals. The traveltime for each of these rays is the sum of the refracted ray traveltime (downgoing and upgoing paths) plus the time taken for a ray to propagate along the boundary from the intersection point to the point of emergence. This approximation means that the exact traveltime and position of a critical refraction are never calculated. But, the approximation is good and can be checked easily. A model 40×350 km with rays separated initially by 1 degree had an error of only 5 msec (Cumming et al. 1979).

EXAMPLES OF USE

We have chosen to illustrate the method with two models. The first is a laterally homogeneous one which has a complex velocity-depth curve which includes strong velocity gradients and a low velocity zone. Using such a model allows a comparison between the traveltimes calculated from our ray tracing procedure and those determined by direct calculation from the velocity-depth values.

Figure 1 shows the velocity-depth curve derived by Malecek and Clowes (1978) during interpretation of marine deep seismic sounding data recorded across Explorer Ridge off Canada's west coast. The interpretation was

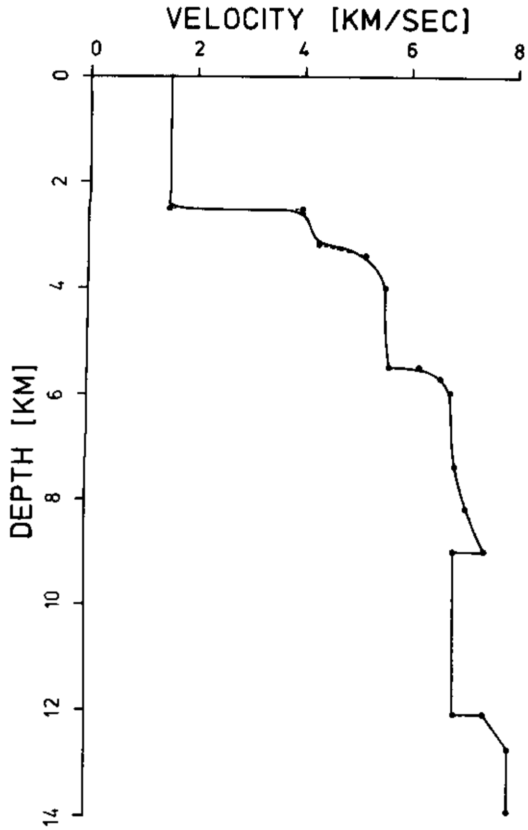

Fig. 1. Continuous velocity-depth curve (solid line) for offshore region (after Malecek and Clowes 1978) and 13-layer approximation (solid dots joined by dashed lines) used for the laterally homogeneous crustal model of Fig. 2.

made to satisfy both the traveltime and amplitude content of the data, and so required the calculation of snythetic seismograms. The disk ray theory (DRT) method described by Wiggins (1976) was used because the algorithm is inexpensive to run and thus many models could be tried. Figure 3a shows the DRT synthetic seismic section and the traveltime curves which are consistent with the smooth, continuous velocity function of Fig. 1.

To establish a model for ray tracing, the continuously varying velocity-depth profile was approximated by 13 boundaries with linear velocity gradients between interfaces, as shown by the velocity-depth profile with straight lines joining dots (Fig. 1). The resultant laterally homogeneous crustal model (Fig. 2) has 12 layers above a half space and extends for the length of the observed seismic profile. The source is located at zero distance and depth. Rays emerging from this point, at angles from 68 to 80 degrees with respect to the horizontal in increments of 0.1 degree, were traced through the model as described previously. These are shown in Fig. 2. Layers with strong velocity gradients are notable for the high curvature associated with the rays passing through them. The low velocity zone between 9 and 12 km depth acts as a mirror for rays emerging from the lower interface at exactly the critical angle. After one reflection from the upper surface, the rays are incident on the lower boundary, again exactly at the critical angle. In our simple formulation, and for the purpose of interpreting the observed seismograms, such rays are not important and their propagation is terminated. This enables us to avoid computing difficulties associated with rays incident at precisely the critical angle.

The traveltime-distance values determined for each ray within the set of rays emerging at the surface are shown in Fig. 3b. The solid lines are derived from the continuous velocity function of Fig. 1. Agreement between the traveltimes calculated from ray tracing and from the smoothly varying velocity function is very good, especially for phases which have strong amplitudes. Differences are less than 0.05 s and most of these discrepancies are due to the approximation of the DRT velocity function by a series of layers with linear velocity gradients. However, the ray tracing results also prominently show the traveltimes of secondary phases which may have extremely small amplitudes. The refracted arrivals along AB (Fig. 3b) are an example which show weakly on the synthetic seismograms (A′B′ on Fig. 3a). However, the refracted phase CD (Fig. 3b) which is from the base of the thin layer with a high velocity gradient from 3.2 to 3.4 km depth, has no direct equivalent on Fig. 3a. The very weak phase C′D′ is related to the rapid change in velocity gradient at a depth of 3.2 km (Fig. 1). Through a combination of the choice of angular increments for rays emerging from the source, the velocity-depth structure and the calculation criteria described in the previous section, no refraction arrival from the top of the thin layer (equivalent to C′D′ on Fig. 3a) is generated. What this points out is that care

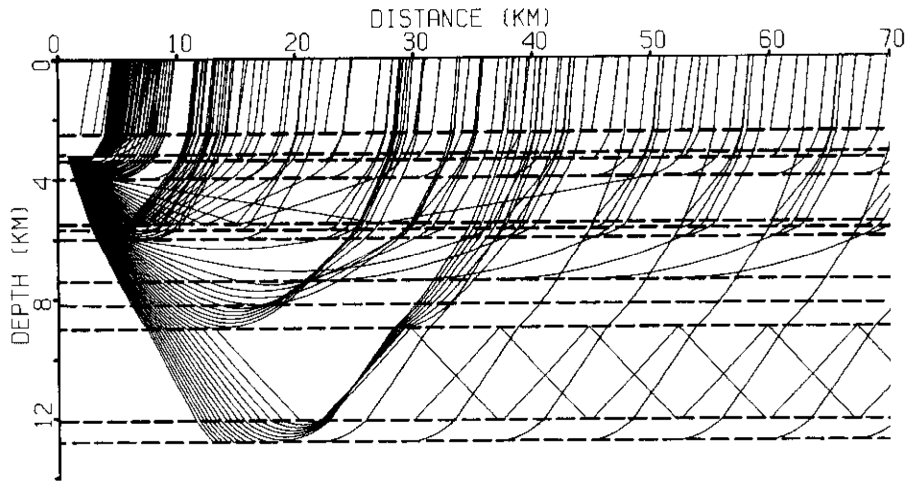

Fig. 2. Laterally homogeneous crustal model (thick dashed lines) based on velocity-depth curve of Fig. 1. Ray paths from the source at 0 km distance and depth are the thin, solid lines. The near-source ray segments have been deleted from the plot.

must be exercised when correlating traveltimes of secondary arrivals from ray tracing with observed traveltimes to ensure that the same phases are being compared.

Qualitative evaluation of amplitude information also can be made from the traveltime-distance values calculated by ray tracing. The cusp at 11.5 km and 3.8 s reduced traveltime and the closely spaced reflection arrivals at greater distances (Fig. 3b) are indicative of large amplitudes and this observation is confirmed by the synthetic seismograms (Fig. 3a). Similarly, cusps and reflection arrivals near 30 km at 3.5 s, and 38 km at 3.6 s, show as large amplitude synthetic seismogram waveforms.

The second model chosen for illustration of the ray tracing algorithm represents a structurally complex crustal cross-section which includes dipping interfaces, vertical faults and individual blocks of material with different velocities. The example was derived during the process of interpreting a complex set of marine crustal seismic data and gives a comparison of observed traveltimes with those determined by ray tracing through the model.

Figure 4 shows one record section compiled from a set of marine seismic data recorded in Winona basin, a deep water sedimentary basin west of northern Vancouver Island (Clowes et al. 1978; Thorleifson 1978). The small triangles show the first arrival picks. Particular features

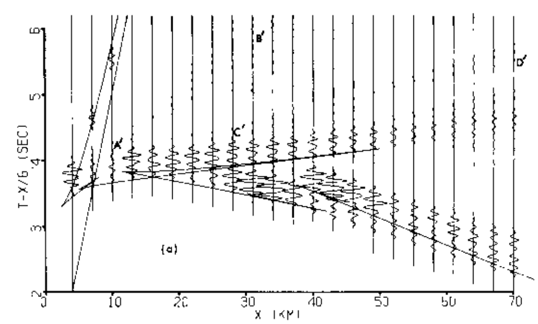

Fig. 3. (a) DRT synthetic seismogram section and traveltime curves (thin, solid lines) corresponding to the continuous velocity-depth curve of Fig. 1.

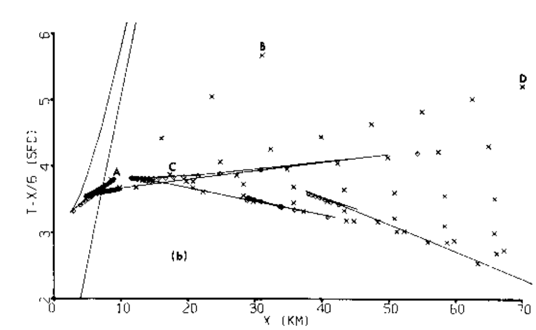

Fig. 3 (b) Traveltime-distance data as determined by ray tracing through the model of Fig. 2. Diamonds are reflection arrivals, X's are critically refracted arrivals. The traveltime curve from (a) is added for comparison. AB and A′B′ coincide, but CD and C′D′ do not (see text).

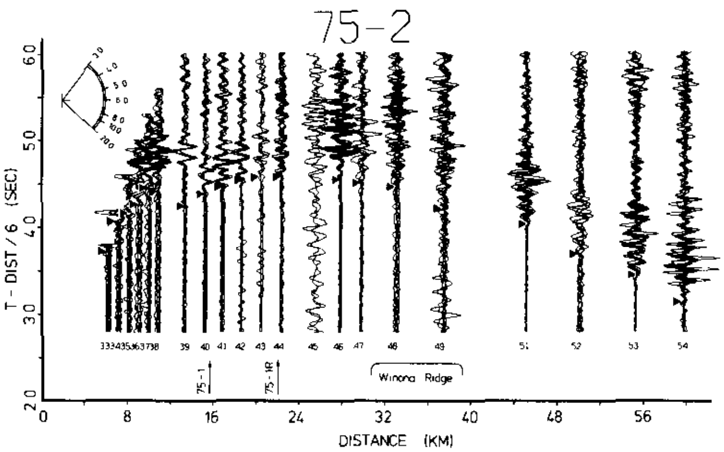

Fig. 4. Reduced record section for profile 75-2 across Winona basin (from Thorleifson 1978). First arrival picks are shown by solid arrow heads. Amplitude corrections have been applied to the seismic traces. A nomogram for apparent velocities in km/s is given in the upper left. Shot numbers are indicated below groups of traces.

are the 0.25 s offset in the traveltimes between shots 37 and 39, the 0.12 s traveltime advance of arrivals which were recorded over Winona ridge, and the high apparent velocity for the first arrivals at distances greater than 40 km. The record section for the reversed profile confirmed the existence of significant lateral variation in structure (Thorleifson 1978). In this case, interpretation techniques such as the DRT synthetic seismogram routine, which are restricted to laterally homogeneous media, cannot be applied. Thus, the ray tracing procedure described in this paper was employed.

A significant constraint for the trial-and-error modelling of the second example was the velocity-depth curve derived for a pair of reversed profiles (75-1 and 75-1R) recorded at right angles to profile 75-2 and parallel to the axis of the basin (Lynch 1977). A second strong constraint was that the resultant model must be such that rays traced through it in the opposite direction give traveltimes consistent with the reversed profile 75-2R.

The crustal model shown in Fig. 5 satisfies these constraints. Within each block outlined by the heavy dashed lines, the velocities were made nearly constant by specifying a negligible velocity gradient. Representing the velocity structure derived from profile 75-1 and 1R by a series of layers with velocity gradients, as we did in the first example, would have unnecessarily complicated the model.

To produce the ray paths plotted in Fig. 5, rays leave the source at X = 0, Z = 0 at angles of 70 to 85 degrees in increments of 0.1 degree. The traveltime-distance data for rays emerging at the surface are shown in the upper part of Fig. 5. First arrival traveltimes from Fig. 4 are given by the solid line through open circles. It is clear that the traveltimes derived by tracing through the model match well with those determined from the record section.

A one kilometer step in the interface between the 2.5 and 4.3 km/s blocks at a distance of 10 km causes an offset in traveltimes which is consistent with the observed offset. Modelling Winona ridge between 30 and 40 km as a separate block with a higher velocity (3.7 km/s) than that of the sediments to either side (2.5 km/s) gives a traveltime advance similar to that observed on the record section. The high apparent velocity of the first arrivals beyond 40 km is introduced by the combined effects of dipping layers. The relatively large amplitudes associated with this phase are consistent with the nearly simultaneous arrival and constructive interference of critical refractions from two or more different interfaces as shown by the overlapping X's on the traveltime-distance plot. The large numbers of secondary arrivals at distances greater than 16 km are consistent with the extended wave trains observed in the data of Fig. 4. However, the principal criterion for the acceptability of the model is the correspondence of first arrival traveltimes.

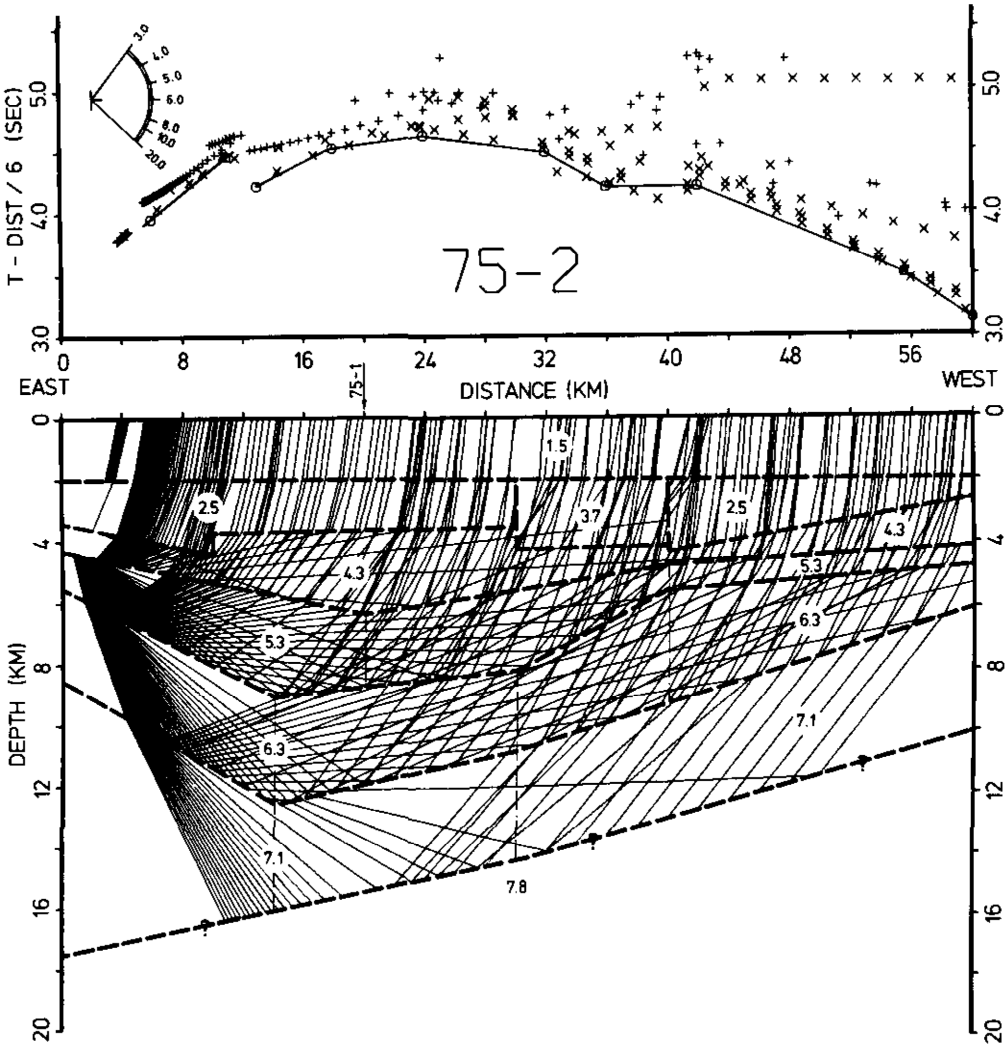

Fig. 5. Upper: Reduced traveltime curves for profile 75-2. The solid line joining open circles represents the first arrival picks taken from the record section of Fig. 4. Plus signs correspond to arrivals of reflected ray paths and X's correspond to arrivals of critically refracted ray paths in the crustal model shown below.

Lower: Crustal model (thick, dashed lines) for profile 75-2; ray paths (thin, solid lines) are superposed. Numbers within blocks are velocities in km/s. The question marks on the bottom interface indicate that the first arrival traveltimes could be satisfied without arrivals from this boundary; therefore its position is not well defined. Vertical exaggeration is 2X.

Discussion

We wish to emphasize that the existence of reversed seismic profiles and the constraints of a reversed pair of profiles perpendicular to 75-2 were essential to the determination of a model as structurally complex as that of Fig. 5. As well, many different perturbations could be made because the computational cost associated with any one model was small. For example, over 230 rays are traced through the crustal model of Fig. 5. Using the Amdahl 470

V/6-II computer at the University of British Columbia, the ray-path calculations for profile 75-2 required about 12 seconds of CPU time. Figure 5 took a total of 20 minutes to plot using the Calcomp plotters. The cost of the entire run was about $4.00 for CPU and $5.00 for plotting. An important factor in expediting interpretation is to preview the plot on a Tektronix graphics terminal. This obviates the need for hard copy plots for each model perturbation and greatly decreases the turnaround time associated with the model.

Persons interested in a copy of the Fortran program are invited to write the authors. A cross-referenced listing, explanatory documentation and examples of input/output files will be included with the card deck.

Conclusion

A fast, inexpensive method for tracing rays through complex, laterally varying structures which have linear velocity gradients in each block has been developed. Ray paths and traveltimes for all reflected and refracted phases are determined. However, multiple reflected ray paths, converted phases and sub-critical incidence reflected arrivals are not considered in this simple approach. Approximate amplitudes may be inferred from the relative numbers of rays emerging at the surface.

The method described in this paper is not intended to be a replacement for the many, more sophisticated theoretical approaches which have been developed. It is a technique designed for practical interpretation. Once a suitable model, consistent with the observed traveltime data, has been obtained using this inexpensive method, the model might be refined by the use of techniques which calculate relative amplitudes of arrivals or generate synthetic seismograms.

Because of the efficiency of the ray path computations, the ray tracing technique described herein probably can be extended to enable the rapid calculation of synthetic seismograms for primary reflected and refracted waves. One approach for doing this might be to combine our ray tracing calculations with the disc ray theory concept of Wiggins (1976) or the WKBJ approximation technique described by Chapman (1978). If such a combination were successful in providing an efficient, inexpensive approach to the generation of synthetic seismograms in laterally inhomogeneous media, then it opens up the possibility of direct inversion of the traveltime and amplitude content of seismograms recorded in a structurally complex region.

Acknowledgements

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