

A DIRECT APPROACH TO REMOVAL OF MULTIPLES †

D. WEHRHAHN* and K. TITCHKOSKY**

ABSTRACT

A new technique for removing multiple energy in seismic data processing has been developed and implemented on a digital computer. The principle used in this technique is based on distinguishing primary and multiple energy by their different velocities. The method involves determining in X the distance where the primary energy is zero and then subtracting this residual multiple energy from the trace sample being processed. Examples of this technique applied to synthetic data, to land data and to marine

data show effective multiple energy cancellation while retaining the original character of the primary energy.

A difference between this process and other multiple attenuation schemes is that no correlation is used to define events. The multiples are discriminated against by their velocity. The user does not have to know the velocities exactly but can input a range of velocities to attenuate multiples and their peg legs for both land and marine seismic data.

INTRODUCTION

Multiples have been a constant problem in seismic exploration. This paper presents a new multichannel technique for the attenuation of multiples, based on the principle that the primary and multiple reflections have different velocities. The process, called ANSWERS, can be applied to land or marine areas where multiples are a problem.

MULTIPLES

Multiple and primary reflections may be defined as follows:

Multiple — All those reflections that have 2 or more upward reflections.

Primary — All those reflections that have only 1 upward reflection.

Figure 1 shows the ray path of a multiple reflection on the left and the path of a primary reflection on the right. The multiple is that of a surface multiple. Multiple reflections can be much more complicated and includes those called 'interbed multiples'. Interbed multiples can give rise to subtle character changes but do not usually produce a discrete event on the seismic record and will not be discussed further in this paper. Multiples are a problem in seismic interpretation because of their interference with the primary reflections. For land this interference can be such that the multiple is mistaken for a primary, or the multiple distorts and shifts the primary reflection, making interpretation difficult. For marine and especially for some deep water areas of the continental shelves this interference is so strong that it simply overpowers the primary reflections, making interpretation impossible. Multiples also adversely affect velocity analysis resulting in poor velocity spectra which are difficult to interpret.

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* Petro Canada Exploration Inc., Calgary, Alberta.

** Aquitaine Company of Canada, Calgary, Alberta.

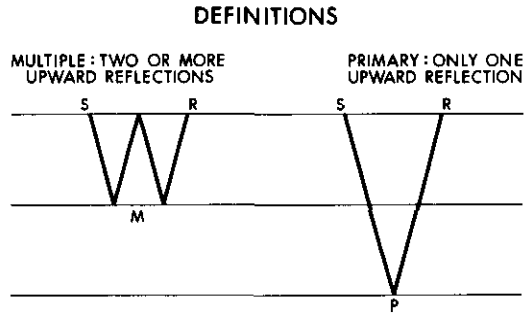


Fig. 1. Definition of Multiple and Primary Reflections.

Multiple interference makes mapping and evaluation of a prospect difficult and can lead to expensive seismic failures. Below are listed some techniques for recognizing multiples.

- 1) Synthetic Seismograms and stratigraphic modelling with and without multiples.
- 2) Autocorrelation.
- 3) Retrocorrelation.
- 4) Anomalous relief (deep events having twice the dip).
- 5) Event repetition on a seismic record.
- 6) General poor quality and variable amplitude and frequency. Of course this does not mean that good looking records are free of multiples.
- 7) Velocity Analysis. The multiple has a different velocity than the primary reflections.
- 8) Stacks at Multiple Velocities. These are stacks with NMO corrections to enhance the multiple reflection.

Knowing that multiples are present is half the battle. The other half is to apply techniques or processes to remove them. Various techniques for multiple attenuation have been used by oil companies and processing centers including ANSWERS, DEWATER, RAMS, WIPER, SUPERSTACK and CDP STACK. The SUPERSTACK algorithm was given at the 1977 SEG meeting in Calgary by Naess. SUPERSTACK and ANSWERS are similar in that neither uses correlation to define events as do many of the

processes listed above. SUPERSTACK uses a statistical approach while ANSWERS uses a deterministic approach. It should be noted that CDP stacking is still the most widely used technique for multiple attenuation for both land and marine data.

Figure 2, shows one type of multiple reflection. This type is common in the North East British Columbia and Rainbow areas. The surface or base of weathering is a good reflector producing a fairly severe multiple problem.

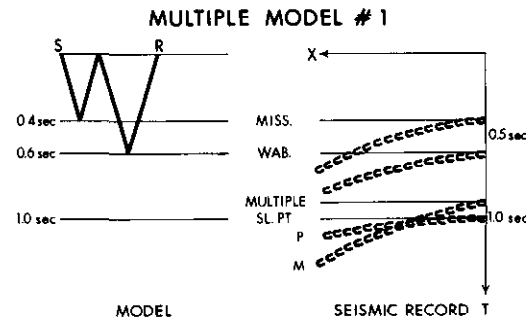


Fig. 2. Land multiple model for North East British Columbia and Rainbow areas.

In Figure 2 shows the multiple whose path is, Mississippian at .4 seconds, to surface then to the Wabamum at .6 seconds producing a multiple at 1.0 second. In this model the multiple results from 2 upward reflections from 2 different layers. Seismic record on the right side of Figure 2, shows the multiple reflection interfering with the Slave Point Reflection. The Slave Point overlays the producing zone. It is important therefore to eliminate the multiple so that the Slave Point event can be mapped correctly. Also as can be seen the multiple reflection on the seismic record has a slower velocity. That is, it has more moveout than the Slave Point primary reflection.

It will be seen that ANSWERS exploits the difference in moveout between the primary and multiple as does CDP stacking.

In Figure 3 shows a typical marine multiple problem. This type of multiple reflection occurs in areas like off-shore Labrador with water depths of 1.5 seconds. The path of the

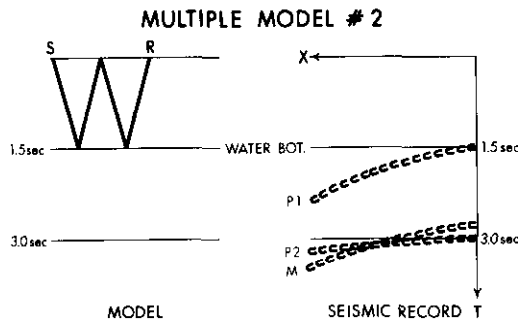


Fig. 3. Marine Multiple Model.

multiple is an upward reflection from the water bottom at 1.5 seconds and then another upward reflection from the same water bottom at 1.5 seconds producing a multiple reflection at 3.0 seconds. The seismic record, shown on the right side of Figure 3, depicts the primary reflection at 3.0 seconds and the multiple reflection just before 3.0 seconds. These long period multiples are quite energetic, since hard water bottoms are excellent reflectors of energy, the surface of the water is almost a perfect reflector (with a reflection coefficient of -1) and there is little attenuation of the energy in the water layer.

This type of multiple reflection is much stronger than a primary reflection occurring at the same time. In this case CDP stacking breaks down and is ineffective in cancelling the multiple by itself. In Figure 3 the multiple is depicted as equal to the amplitude of the primary. In real data cases the deep water bottom multiple can be 10 to 20 times the amplitude of a primary reflection at the same time. It is the case that ANSWERS can handle this for as we shall see it uses the multiple on a near adjacent trace to cancel the multiple on the trace being processed.

ANSWERS TECHNIQUE

The technique directly transforms a set of samples in X which contain both primary and multiple energy to a new set that contain only primary energy.

To describe the algorithm which works sample by sample for all traces we make two observations. First, the primary and multiple energy is additive. Second the primary and multiple reflections have different velocities.

In Figure 4 the primary energy has a frequency of 25 cps. We also suppose there is some multiple energy. The seismic trace is the additive total shown on the right in Figure 4, that is:

$$\text{TRACE} = P + M$$

Now before showing how the velocity or moveout differences will be exploited we make the following observation.

The primary reflection energy is zero when it crosses the time axis. At these points in time the trace value is made up of only multiple energy as the primary is zero. It should also be noted that from a peak to zero crossing is $\frac{1}{4}$ cycle, that is 10 ms for a frequency of 25 cps.

MULTIPLE AND PRIMARY ENERGY IS ADDITIVE

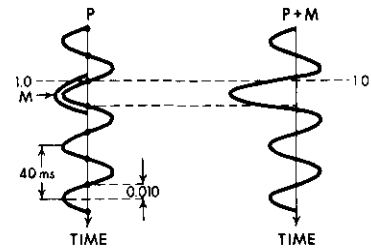


Fig. 4. Multiple and Primary Energy is Additive.

Figure 4 shows the zeros of the primary energy as large dots. It should be noted that it is the seismic trace P + M that is recorded in the field. We wish to remove the multiple energy while retaining the character of the primary. Random noise has not been modeled, for as we shall see, the calculation of the multiple energy M is based solely on a deterministic equation involving the primary velocity V_p and the multiple velocity V_m .

We now describe the ANSWERS algorithm. Figure 5, shows a seismic record or CDP gather with multiple and primary energy recorded. The multiple reflections sweep across the record at a velocity V_m . They interfere with the primary reflection at a velocity V_p .

Consider the trace DA and the sample at time

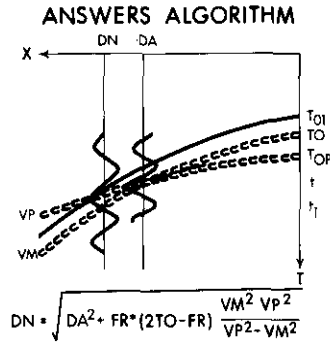


Fig. 5. The ANSWERS technique.

t, this sample value is made up of a multiple part M and a primary part P. The technique is to search forward in the X direction along the multiple reflection to a near adjacent trace DN where the primary has a zero crossing, that is $P = 0$. The distance DN that we must move along is such that the moveout difference between multiple and primary reflection be $\frac{1}{4}$ cycle, that is $\frac{1}{4}$ the period of the primary frequency. The formula for DN is shown at the bottom of Figure 5. FR is the time of $\frac{1}{4}$ cycle for the primary frequency. TO is as shown in Figure 5. The equation for DN is based solely on the NMO equations for the multiple and primary reflections. The derivation is given in the appendix.

Having found DN we then obtain M' at DN. Now assuming that the multiple does not vary much from DA to DN which is a reasonable assumption, that is $M' = M$: we now subtract the M' value from $P + M$ leaving only the primary energy at Trace DA, sample t. We then do the same for the next sample on Trace DA. When Trace DA is completely processed we start on the next trace. This continues until all traces are processed.

An interesting variation of the above algorithm is to search forward $\frac{1}{4}$ cycle to get $S_F = M - P$ and search back $\frac{1}{4}$ cycle to get $S_B = M + P$, then $M = (S_F + S_B)/2$.

There are very few input parameters required for this process. In Figure 6 we list the input parameters. They are TS which is the time on trace where the multiple cancellation should start, TF which is the time on the trace where the multiple

1. TS START TIME
2. TF END TIME
3. VM MULTIPLE VELOCITY
4. VP PRIMARY VELOCITY
5. FR $\frac{1}{4}$ CYCLE TIME

Fig. 6. User input parameters.

cancellation is to finish, and Vm is the multiple velocities to remove. The process does not require the multiple velocity to be uniquely specified but a range can be given, for example, 4800'/sec. to 5000'/sec. Vp is the primary velocity function and this should be a best estimate but does not require the accuracy of the NMO velocity. FR is $\frac{1}{4}$ the cycle time of the primary reflection.

SYNTHETIC EXAMPLES

In Figure 7, shows a seismic record or CDP gather of 24 traces. The trace spacing is 100 metres. There are 2 primary reflections P1 and P2 at a velocity of 2100m/s. Reflection P2 has the polarity opposite to P1. There are 2 multiple reflections M1 and M2 cutting across the primaries at the velocity of water 1500m/sec. In fact M1 cuts across both P1 and P2.

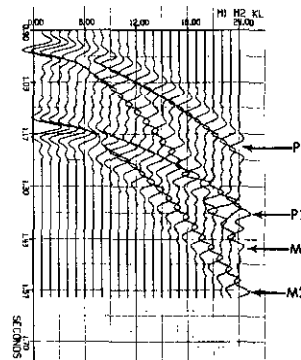


Fig. 7. Marine Synthetic Seismic Record BEFORE ANSWERS.

In Figure 8, shows the 24 traces after ANSWERS. As can be seen the multiple energy has been removed while retaining the primary reflections without distortion. The wavelet used in these synthetics was a Klauder Wavelet.

is 11000'/s and the multiple velocity is 8000'/s. The multiple was taken to be 2.5 times the size of the primary. The noise energy was taken equal to the primary energy. The wavelet in this example is a 30 Hz Ricker Wavelet.

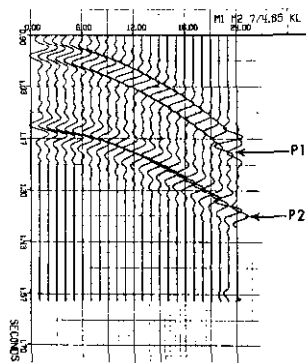


Fig. 8. Marine Synthetic Seismic Record AFTER ANSWERS.

A synthetic example which is representative of land multiple problem is presented in Figure 9, which shows a seismic record of 24 traces with a primary reflection P1, a multiple reflection M1, and some noise. Its a bit difficult to see the primary reflection but those geophysicists accustomed to working with poor land data will have no trouble. The trace spacing is 220' and the primary velocity

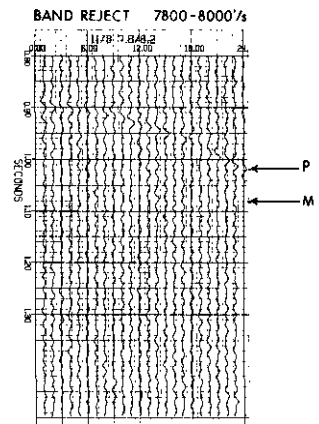


Fig. 10. Land Synthetic Seismic Record — AFTER ANSWERS.

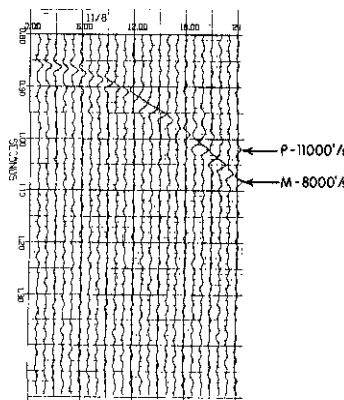


Fig. 9. Land Synthetic Seismic Record — BEFORE ANSWERS.

Figure 10 shows the 24 traces after ANSWERS. The multiple has been eliminated while the primary is retained. The noise of course remains and it will be left to CDP stacking of the data to improve the S/N ratio. This multiple was cancelled using a multiple velocity range of 7800 - 8200'/s.

ACTUAL DATA EXAMPLES

Examples of ANSWERS applications to actual field data are given below. First example consists of some 6 fold land data from the Rainbow area.

Comparison of the velocity analysis at SP 813 for a line in the Rainbow area is shown in Figure 11. On the left side we have the analysis before ANSWERS. The vertical scale is in time varying from 100ms. to 2000ms. The horizontal scale is in velocity varying from 7000'/s to 13000'/s. On the right side we have the same velocity analysis run with identical parameters except done after ANSWERS. ANSWERS was run to reject multiple velocities 7800'/s - 8200'/s.

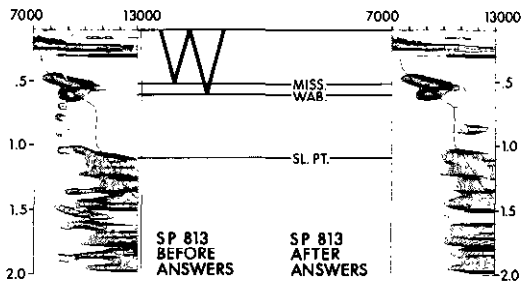


Fig. 11. Rainbow Velocity Analysis — Before and After ANSWERS.

Superimposed on both velocity spectra is the velocity curve of a well drilled on the seismic line through the reef. In this case ANSWERS is applied to a single dip optimized gather at the velocity analysis point and takes very little computer time. The Rainbow field is a fairly old oil field but as demonstrated by Pembina one can find more oil in old fields using new techniques in seismic exploration.

The significant problem in this area is caused by the MISSISSIPPIAN-WABAMUM multiple. This multiple occurs at about 1.0 second and interferes with the Slave Point reflection which overlays the Keg River Reefs. On the BEFORE spectra the multiples are clearly visible at .7, .8 and .9 sec. Also the MISS-WAB multiple manifests itself at 1.0 second as a long tail reaching toward the lower velocities.

On the AFTER ANSWERS spectra the multiples at .7, .8 and .9 have been removed. Further, a primary reflection pick is now clearly visible at .7 seconds. This pick also agrees with the well velocity curve. Also the semblance of the pick at .9 seconds is much higher, and the tail at 1.0 seconds which corresponded to the multiple at the Slave Point has also been removed. The importance of applying ANSWERS to enhance velocity analysis cannot be over emphasized. It can improve CDP stacking to such an extent that applying ANSWERS to the entire line is no longer necessary. In this case we did apply ANSWERS to the entire line.

Figure 12, shows the stack sections Before ANSWERS. In Figure 13, we show the stack section After ANSWERS. Both sections were stacked using the well velocity curve shown

in Figure 11. The ANSWERS process was applied directly to the 24 trace 100% shotpoints.

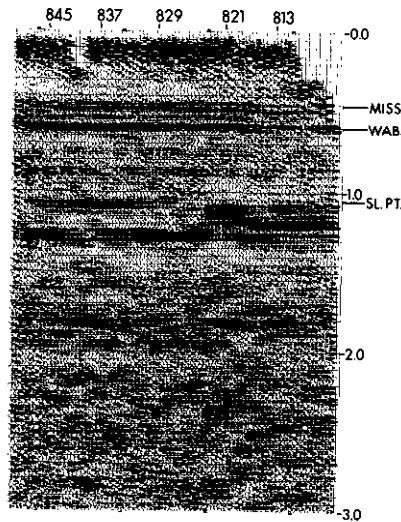


Fig. 12. Rainbow Stacked Section — BEFORE ANSWERS.

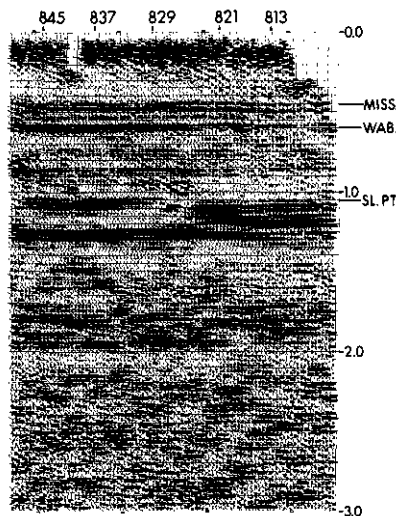


Fig. 13. Rainbow Stacked Section — AFTER ANSWERS

The ANSWERS stack shows improved reflections in the critical zone of 1.0 - 1.3 seconds for the entire line. This is especially true of the Black Creek salt reflection

beginning at 1170 ms at SP 813 and going right up to the reef at SP 829 at 1210 ms. Also the Slave Point reflection above the reef at 1060 ms from SP 829 to SP 851 is more consistent on the ANSWERS stack. This is also true of the Slave Point reflection off reef (SP 813-829) dipping from 1060 to 1080 ms. The interpretation of this line would probably be the same whether the ANSWERS stack was used or not. It must be noted however that this line was shot using a spread specially designed for the MISS-WAB multiple at the Slave Point level. It is for this reason the stack is quite effective even without ANSWERS. Of course, not all seismic data was recorded with the specially designed spread and here the ANSWERS stack should show considerable improvement over the stack without ANSWERS. Further, ANSWERS could be effectively applied to remove multiples from old seismic data shot single fold. In this way a prospect may be evaluated using old seismic data to bring out primary reflections that were previously buried in multiple reflections. In the Rainbow example, we also suggest that measurements of transit time from Slave Point to Red Beds at 1280 ms to estimate porosity of the reef will be more accurate on the ANSWERS stack.

We now discuss an example of applying ANSWERS to marine data to cancel water bottom multiples and their peg-legs. This example is taken from Offshore Labrador and was recorded 48 fold with a trace spacing of 100 meters.

In Figure 14, we show one 48 fold CDP gather before and after ANSWERS.

This is CDP 53 on the line and the water bottom is at 1.5 seconds here. This amounts to a water depth of 3500 feet or about 1000 meters at this east end of the line. This line runs east to west and the water depth decreases as we go west toward the shore. The first water bottom multiple is quite strong and occurs at 2.950 seconds, followed by strong peg-legs.

The right side of Figure 14, shows the same CDP after the multiple removal process was applied to it. The multiple velocity band rejected for this CDP gather was from 4500'/s - 4900'/s. As can be seen, the water bottom multiple has been effectively attenuated.

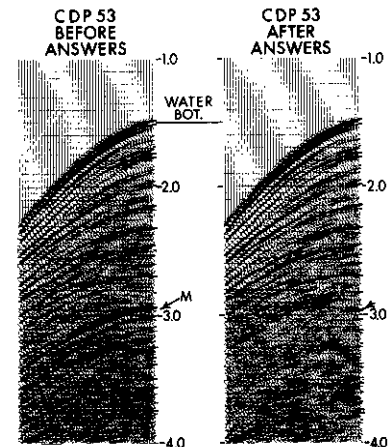


Fig. 14. Offshore Labrador 48 fold CDP Gathers — Deep Water.

Also, the peg-legs immediately following the multiple reflection at 2.950 have also been attenuated.

To further demonstrate the effectiveness of this multiple removal technique we did a velocity analysis. In Figure 15, we compare the velocity analysis at this CDP before and after ANSWERS. The water bottom multiple is clearly visible at 2.9 seconds. In fact after the multiple reflection at 2.9 sec. there appears to be very little primary reflection. On the right side of Figure 15, we show the velocity analysis obtained with identical parameters except the multiple removal process was applied before velocity analysis was done. The multiple at 2.9 seconds has been effectively removed. Further primary energy is now clearly visible at 3.3s (7400'/s) and at 3.8 sec. (8100'/s). We now apply ANSWERS to the West end of this seismic line where the water depth is less.

Figure 16 shows CDP 644 Before and After ANSWERS. This 48 fold CDP gather is from near the West end of the line. Here the water depth is 1650' or 500 meters. The water bottom reflection is at .65 seconds for this CDP. One can clearly observe the strong water bottom multiples, M1 at 1.3, M2 at 1.9 and M3 at 2.5 seconds on the gather before ANSWERS on the left side of Figure 16. One also observes a primary reflection at 1.7 seconds situated between the multiples M1 and M2.

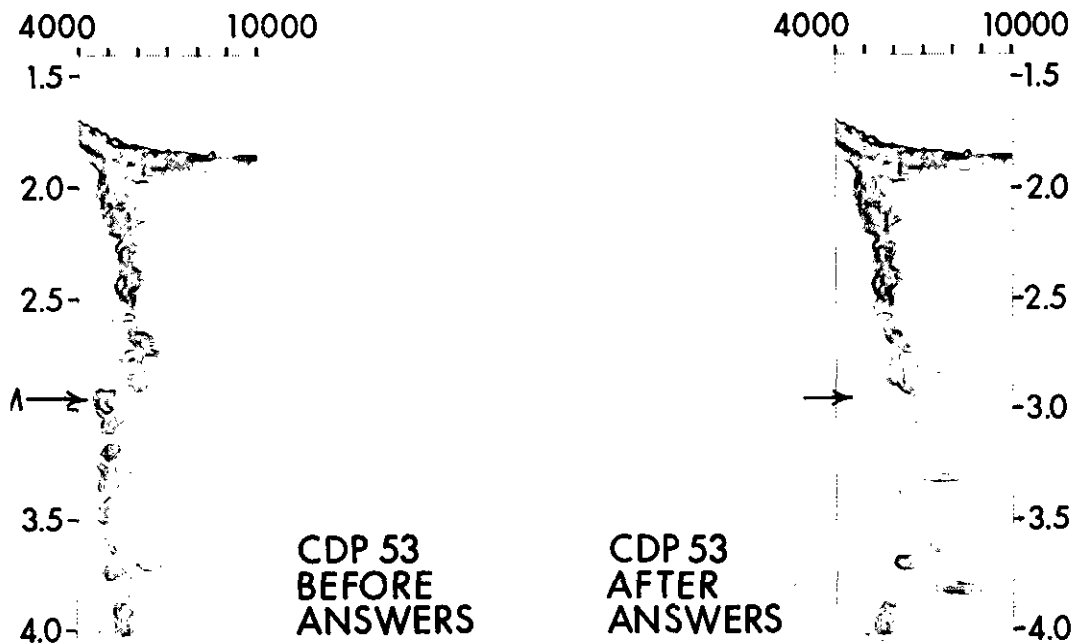


Fig. 15. Offshore Labrador Velocity Analysis — Deep Water.

On the right side of Figure 16, same CDP is shown after the multiple removal process was applied to it. The band of velocities rejected were from 4800'/s to 5000'/s. The 3 water bounce multiples M1, M2 and M3 have been

effectively attenuated. The primary reflection at 1.7 seconds and the less obvious primary reflection at 1.4 seconds remain.

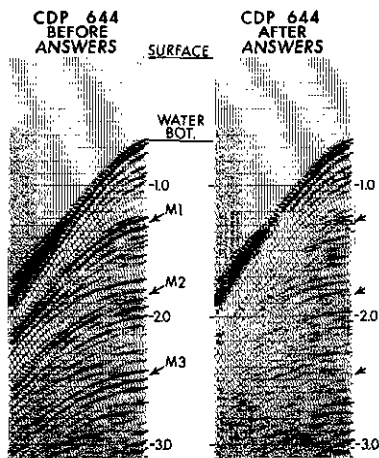


Fig. 16. Offshore Labrador 48 fold Gathers — Shallow Water.

In Figure 17, we compare the velocity analysis at this CDP with and without ANSWERS. The left side of Figure 18 shows the velocity analysis without ANSWERS. The water bottom multiples at 1.3, 1.9 and 2.5 seconds with velocities of about 4800'/s are clearly visible. Further one observes a primary reflection at 1.2 seconds but its semblance is only average. The right side of Figure 17 shows the velocity analysis done with exactly the same parameters except that velocity analysis was done after ANSWERS.

For this analysis the water bottom multiples at 1.3, 1.9 and 2.5 seconds are clearly removed. Further, the semblance of the primary reflection at 1.2 seconds is greatly improved, and a new primary reflection is now clearly pickable at 1.4 seconds. In the analysis without ANSWERS the water bottom multiple at 1.3 seconds was so over-powering that the primary reflection at 1.4 seconds did not come through.

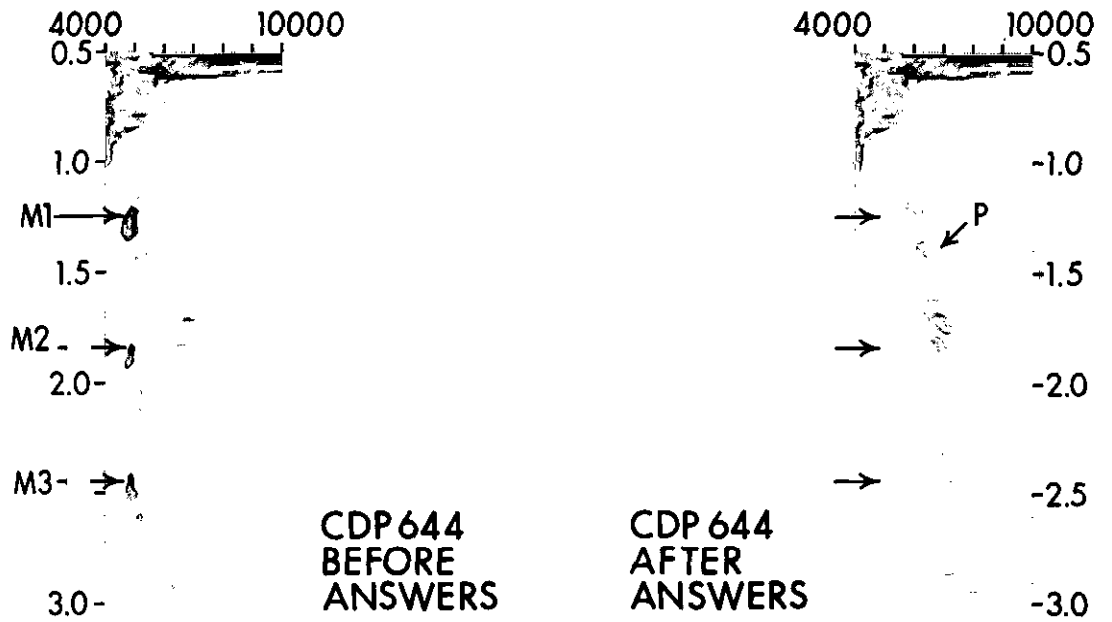


Fig. 17. Offshore Labrador Velocity Analysis — Shallow Water.

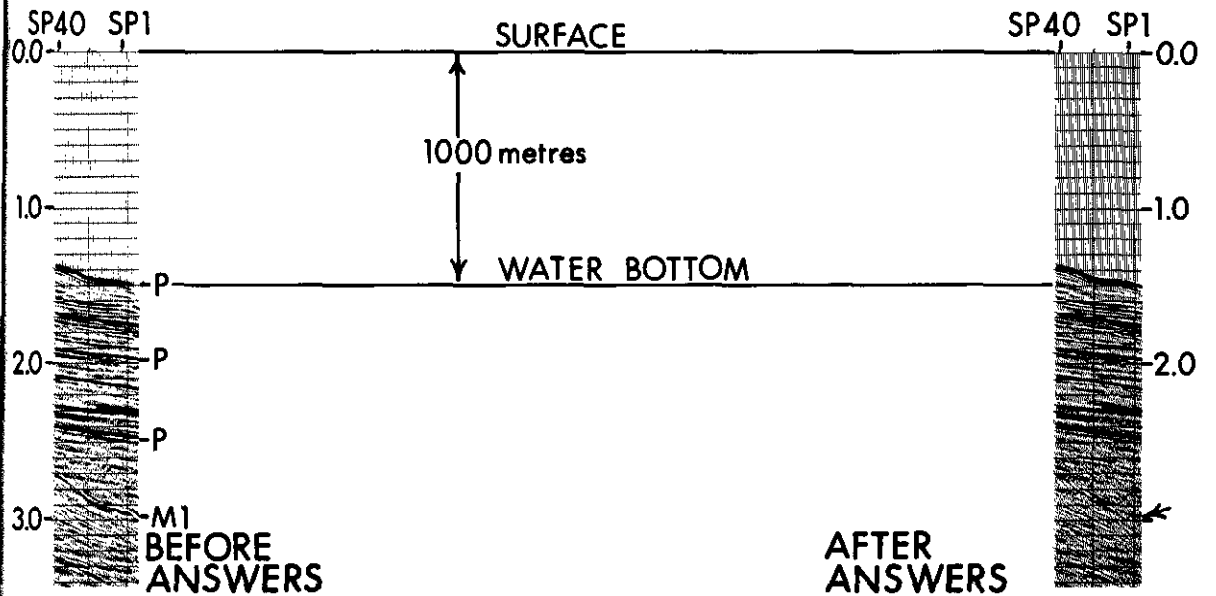


Fig. 18. Offshore Labrador Stacked Section in Deep Water Area.

The final example is to compare the actual stack section Before and After ANSWERS.

In Figure 18, shows a portion of the stack section in the deep water region with and without ANSWERS. The left side of Figure 18 shows the 48 fold stack section before ANSWERS. One can see the water bottom multiple snaking across the section at 2.9 seconds. Additional peg-legs below this multiple reflection at 2.9 seconds are also visible.

The right side of Figure 18 shows the 48 fold stack section with ANSWERS. All other processing parameters were identical. The multiple attenuation or dereverberation is not perfect in that some multiple remains at about SP 20 where there is a curved depression in the water bottom.

At this end of the line the multiple problem is not serious as it is not interfering with any visible primary reflections. This is not the case at the West end of the line where we have only about 1650' or 500 meters of water.

Figure 19, shows a portion of the stack section in the shallow water portion. On the left side of Figure 19 we show the 48 fold stack section before ANSWERS. One can see the water bottom multiples cutting across at 1.3 and 1.9 seconds. This is quite serious as they are interfering with the primary reflections. This interference will make interpretation and mapping these primary reflections difficult.

On the right side of Figure 19 we show the same stacked portion with ANSWERS applied before stacking. The water bottom multiples at 1.3 and 1.9 seconds have been removed while the primary reflections retain their original character. It should be noted that the sequence of processing was DMX - SORT - ANSWERS - DECONVOLUTION - NMO - STACK - FILTER.

In Figure 20, we show the entire stack section of the offshore Labrador line before ANSWERS and Figure 21, shows this same section after ANSWERS. As can be seen the multiples have been substantially removed

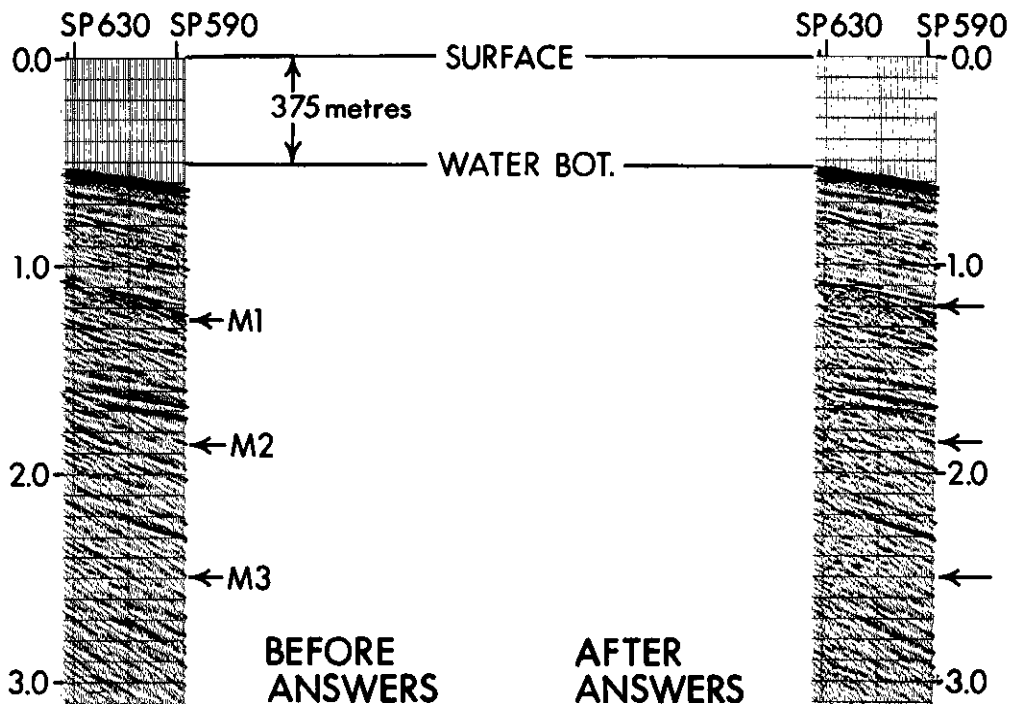


Fig. 19. Offshore Labrador Stacked Section in Shallow Water Area.

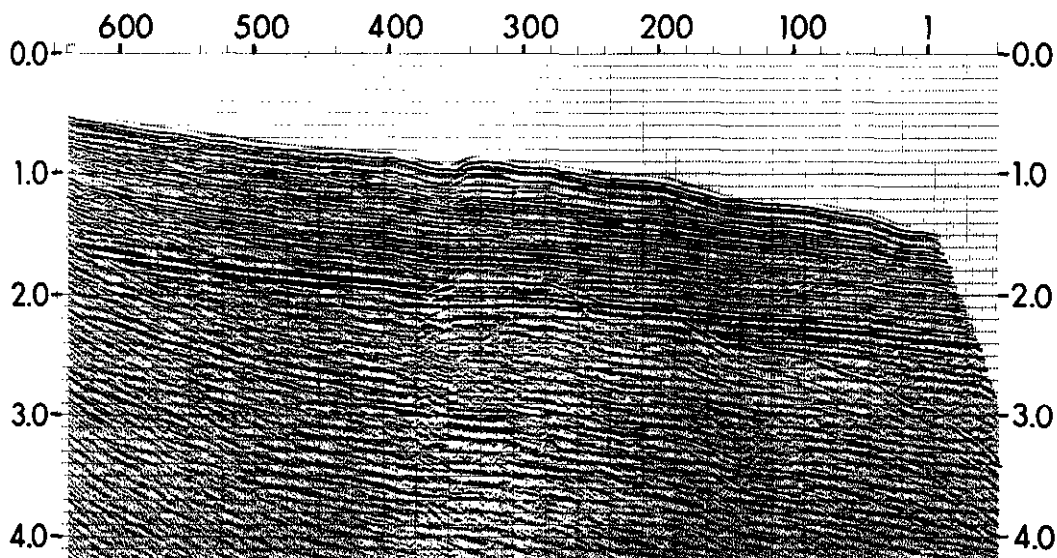


Fig. 20. Offshore Labrador Entire Stacked Section — BEFORE ANSWERS.

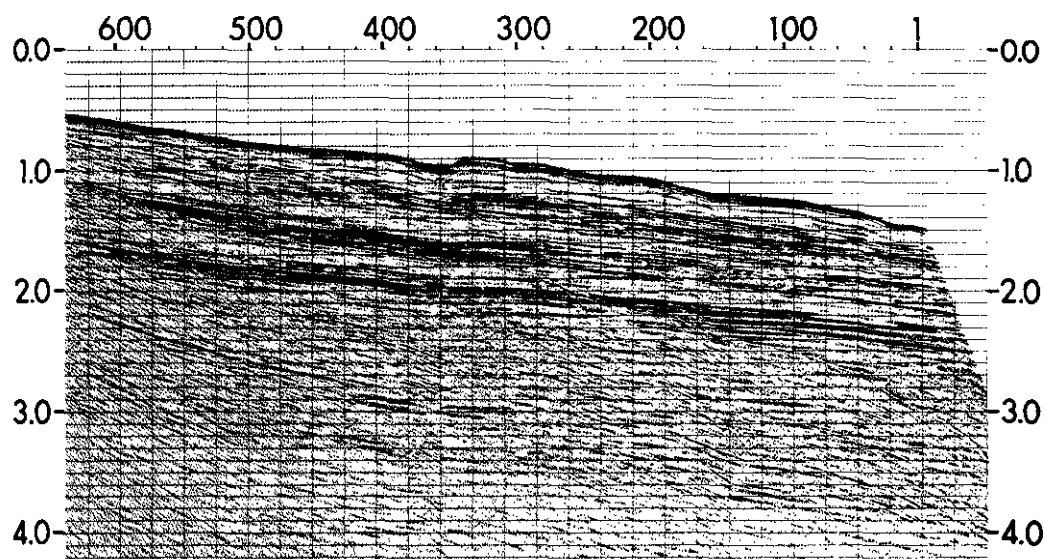


Fig. 21. Offshore Labrador Entire Stacked Section — AFTER ANSWERS.

making mapping of the primary reflections more precise.

The final example (Figure 22) shows a stack section from the offshore Arctic islands. The multiples are extremely strong on the right side from the point on the line where

the Mesozoic and Paleozoic sections meet. Figure 23, shows this stack with ANSWERS processing. There is considerable improvement with the severe multiples mostly removed on the right side of the section. The primary reflections on the left side of the section show improved continuity.

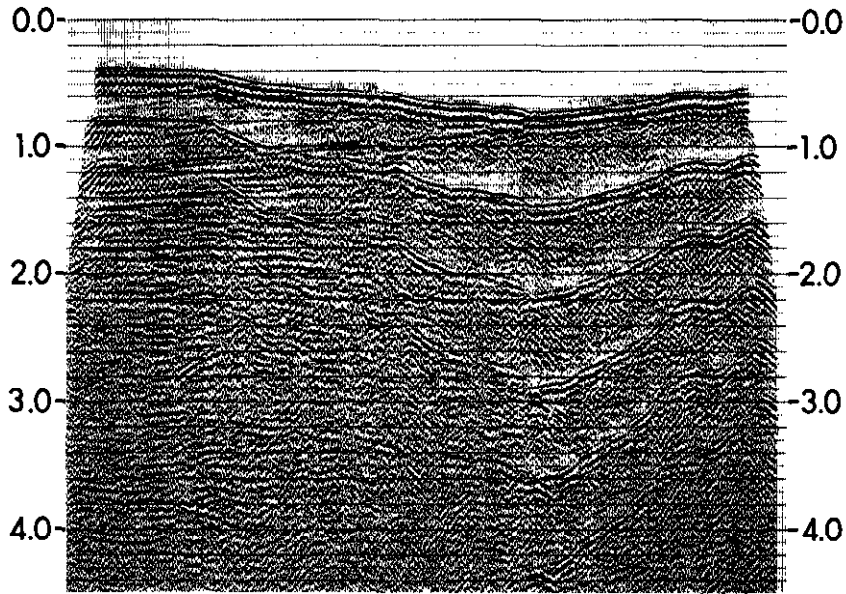


Fig. 22. Arctic Islands 12 Fold Stacked Section — BEFORE ANSWERS.

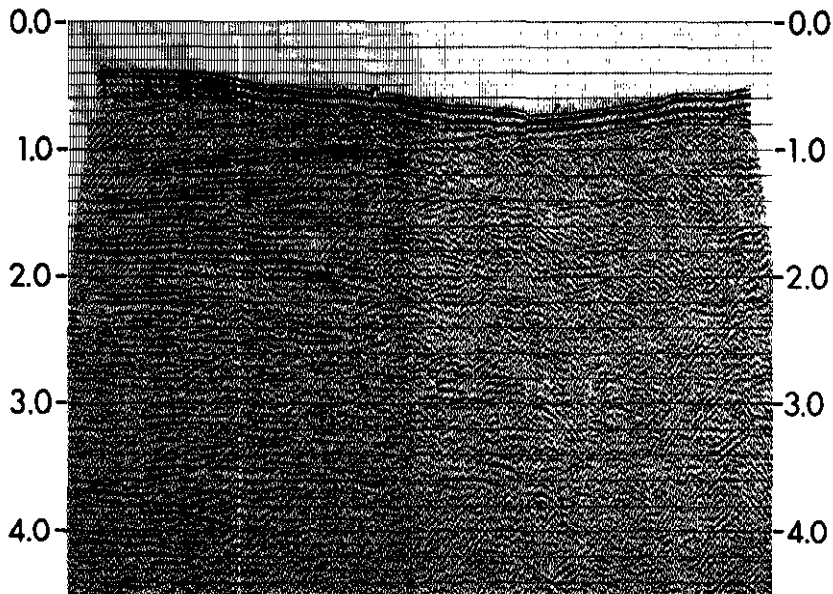


Fig. 23. Arctic Islands 12 Fold Stacked Section — AFTER ANSWERS.

The characteristics of the ANSWERS multiple removal process are summarized below:

- 1) It removes the multiples at the user specified velocities. The exact velocity of the multiple does not have to be specified. Only a range of velocities that includes the multiple to be removed is required to be given by the user.
- 2) Primary reflections are not attenuated and retain their character.
- 3) The process is not effected by random noise.
- 4) No correlation is used to define events.
- 5) It applies to land multiples and to marine multiples which are so strong at times that even CDP stacking breaks down.
- 6) It is like CDP stacking in that it exploits the moveout differences of the primary and multiple reflections but the data is not stacked in ANSWERS.
- 7) It improves velocity analysis, by diminishing the multiples.
- 8) It makes CDP stacking more effective and CDP stacking is still the most powerful and widely used technique today. CDP stacking is more effective because of improved velocity analysis and the multiple energy is considerably attenuated before stack.
- 9) It applies to old seismic data as well as to the most recently recorded data. In fact it could be applied to pre-tape single fold data to cancel multiples to evaluate prospects at a considerable saving of exploration dollars.
- 10) It runs on an IBM 360/44 at about 1 minute per record (24 traces). This is with no optimization and no utilization of the array processor.

CONCLUSION

To sum up, multiple removal process presented here can be applied to both land

and marine data. The process removes multiples which have been a constant problem in seismic exploration. This results in improved velocity analysis and improved stacking allowing improved mapping and evaluation of a prospect. The ANSWERS process does not replace stacking but it works with CDP stacking making CDP stacking more effective. Further, we believe it can be effectively applied to old and new seismic data wherever multiples are a problem.

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APPENDIX

Using the notation used in Figure 5 we have the following NMO equations:

$$\left. \begin{aligned} 1) \quad T_{MA}^2 &= T_0^2 + \frac{DA^2}{VM^2} \\ 2) \quad T_{MN}^2 &= T_{01}^2 + \frac{DN^2}{VM^2} \end{aligned} \right\} \text{Multiple equations}$$

$$\left. \begin{aligned} 3) \quad T_{PA}^2 &= T_{0P}^2 + \frac{DA^2}{VP^2} \\ 4) \quad T_{PN}^2 &= T_{0P}^2 + \frac{DN^2}{VP^2} \end{aligned} \right\} \text{Primary equations}$$

We have equation 1 = 3 and 2 = 4 as $T_{MA} = T_{PA}$ and $T_{MN} = T_{PN}$.

$$a) \quad T_0^2 + \frac{DA^2}{VM^2} = T_{0P}^2 + \frac{DA^2}{VP^2}$$

$$b) \quad T_{01}^2 + \frac{DN^2}{VM^2} = T_{0P}^2 + \frac{DN^2}{VP^2}$$

Subtract a - b and we get:

$$T_0^2 + \frac{DA^2}{VM^2} - T_{01}^2 - \frac{DN^2}{VM^2} = \frac{DA^2}{VP^2} - \frac{DN^2}{VP^2}$$

$$DN^2 \left(\frac{1}{VM^2} - \frac{1}{VP^2} \right) = DA^2 \left(\frac{1}{VM^2} - \frac{1}{VP^2} \right) + T_0^2 - T_{01}^2$$

$$DN^2 = DA^2 + (T_0 - T_{01})(T_0 + T_{01}) \frac{VM^2 VP^2}{VP^2 - VM^2}$$

Now $T_0 - T_{01}$ is approximated to the $\frac{1}{4}$ cycle time difference FR.

$$\text{That is } T_0 - T_{01} = FR \quad \begin{array}{l} T_{01} = T_0 - FR \\ \text{so } T_0 + T_{01} = 2T_0 - FR \end{array}$$

$$\text{This gives } DN^2 = DA^2 + FR (2T_0 - FR) \frac{VP^2 VM^2}{VP^2 - VM^2}$$

$$DN = \sqrt{DA^2 + FR (2T_0 - FR) \frac{VP^2 VM^2}{VP^2 - VM^2}}$$

Which is formula for the distance where the moveout time difference is $\frac{1}{4}$ the period of the basic primary frequency.