A NOTE ON SYNTHETIC VIBROSEIS SWEEP GENERATION

by

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In reading through the article "The Vibroseis System of Seismic Mapping" by Mr. R. L. Geyer (J.C.S.E.G. 6, 1, 1970) I came across some equations (see Fig. 9 in Geyer's article) which did not follow algebraically one from the other, namely:

$$S=Re(Rect(t/T)\ \exp(2\pi i(f_0 t+kt))) \quad (1)$$

$$S=t/T\ \cos(2\pi(f_0 t+kt^2/2)) \quad (2)$$

$$=\cos(2\pi f_0 t)\ \cos(\pi kt^2)-\sin(2\pi f_0 t)\ \sin(\pi kt^2) \quad (3)$$

These equations do not follow logically one from the other and in looking at them I was led to reconstruct that section of his article.

I would like then to start with the expression given by Klauder et al[1] for a general linearly-swept-frequency waveform

$$y(t) = Rect(t/T)\ \exp\ (2\pi i(f_0 t+ kt^2/2)) \quad (4)$$

$$\text{where } Rect(t/T) = \begin{cases} 1 \text{ if } T/2\leqslant t \leqslant T/2 \\ 0 \text{ if } T/2> t > T/2 \end{cases} \quad (5)$$

with T = the duration of the pulse

and where f_0 and k are still to be defined but f_0 is a reference frequency and k is a rate of increase of frequency with time. Now the real part of (4) is clearly

$$Rect(t/T)\ \cos 2\pi(f_0\, t + kt^2/2) \quad (6)$$

and not as given in (2). We may replace the modulating function Rect(t/T) by any suitable function, let us call it A(t) and leave it undefined for the moment except to say that it exists on the interval $-T/2\leqslant t>T/2$. We may write, for the general linearly-swept-frequency signal

$$y(t) = A(t)\ \cos 2\pi(f_0\, t + kt^2) = A(t)\ \cos 2\pi f(t) \quad (7)$$

with $f(t) = f_0\, t + kt^2/2$ and then define the instantaneous frequency

$$f_i(t) = \ = \partial f/\partial t = f_0 + kt. \quad (8)$$

We note that in defining Rect(t/T) in the original equation (6) we assumed that the sweep was of length T in time, extending over the

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interval $-T/2 < t > T/2$ and we shall use this interval to sweep the signal in frequency from a low frequency f_1 at $t = -T/2$ to f_2 at $t = T/2$, $f_1 < f_2$.

Thus equn. (8) at $t = -T/2$ reads

$$f_1 = f_0 - kT/2 \qquad (9)$$

while at $t = T/2$ it reads

$$f_2 = f_0 + kT/2 \qquad (10)$$

so that $k = (f_2 - f_1)/T$ — and $f_0 = (f_2 + f_1)/2$

Then equation (7) becomes

$$y(t) = A(t) \cos(\pi t/T \; (f_2 \; (t+T) - f_1 \; (t - T))) \qquad (11)$$

It should be noted that at $t = T/2$ the function f has the magnitude

$$f = t/2T(f_2 \; (t + T) - f_1 \; (t - T)) \rightarrow T/8 \; (3f_2 + f_1) \qquad (12)$$

and does not equal f_2; it is only the instantaneous frequency which is equal to f_2 at $t = T/2$.

One may now compute the expression $y(t)$ by letting $\triangle t =$ the sample period of the data and let $t = n\triangle t$, $T = m\triangle t$ so that

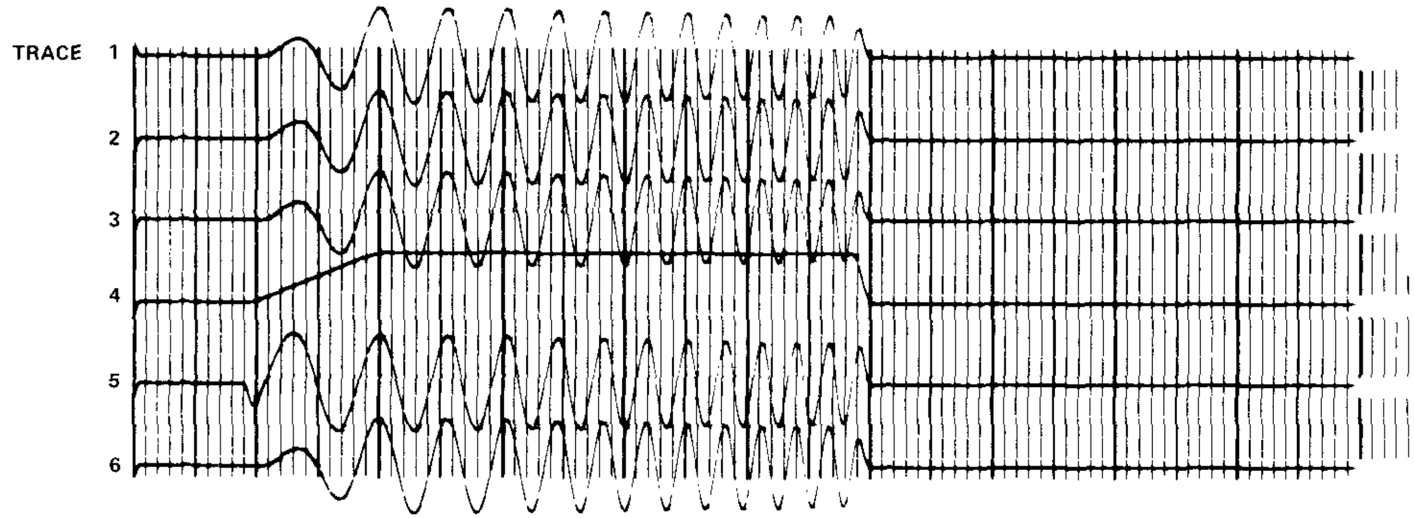
$$y(n\triangle t) = A(n\triangle t) \cos \; (\pi n/m \; (f_2 \; (n + m) - f_1 \; (n - m)) \triangle t) \qquad (13)$$

which of course is to be computed for $-m/2 \leqslant n \leqslant m/2$.

Some typical sweeps generated by use of (13) are shown below, and the modulating function $A(t)$ is also shown, it has been chosen to simulate the rise and fall time of a Vibroseis sweep.

References:— (1) "The theory and Design of Chirp Radars"

Bell System Tech. Journ. 39, 4, 1960



Trace 5 unmodulated L.S.F. pulse $f_1 = 10$, $f_2 = 40$ Hz.

Trace 4 modulating envelope.

Traces 1, 2, 3, 6 final modulated L.S.F. pulse.

(Note. Decrease in overall amplitude of the sweep at higher frequencies is due to plotter attenuation)

$\Delta t = 4$ m.s. Sweep duration 500 m.s.

Ramp on 100 m.s. Ramp off 10 m.s. Delay 100 m.s.