Stacking and Interval Velocities in a Medium with Laterally Inhomogeneous Layers

E. Blias*
Revolution Geoservices, 1102, 733 - 14 Ave SW, Calgary, AB, T2R 0N3
emilb@shaw.ca

ABSTRACT

Depth velocity models are an essential part of seismic data processing and interpretation. They are used in time-to-depth migration and transformation, AVO analysis and pore-pressure prediction. Explicit analytical formulae for near-zero-offset traveltime play a significant role in the seismic method. These formulas allow us to understand the connection between the velocity model and the NMO function. They are very useful if they have a clear physical meaning and can be interpreted in terms of the velocity model and the NMO velocity. These explicit presentations of traveltime for close-to-zero offset are very important in Dix-type traveltime inversion. Dix's type inversion formulae were considered by different authors (Chernjak and Gritsenko, 1979, Goldin, 1986, Krey and Hubral, 1980, last two books contain many references). These formulas were derived for the locally-homogeneous layers. It means that the interval velocities do not change within NMO offset interval or we can neglect these changes. However, as analytically shown by Blias (1981, 1987, 1988), nonlinear lateral changes in shallow interval velocities can affect NMO velocity and often should be taken into account.

In this paper, I analytically consider the behaviour of the stacking velocities in a medium with an overburden velocity anomaly. The influence of the laterally inhomogeneous overburden layer on the stacking velocities can be investigated using ray tracing for some specific models. It's interesting, that in spite of many results, for horizontally inhomogeneous medium our knowledge about its general properties is small.

Using a method, developed by Blias et al., (1984) and Blias (1985), the explicit formulae, connecting laterally changing interval velocities to stacking velocities, have been obtained. These formulae show that we can establish a linear connection between second derivatives of the shallow interval velocities and the stacking velocities for deep horizontal layers. These formulae have a form which has a clear physical meaning and can be given a clear interpretation. The new Dix-type formulae for the traveltime inversion have been derived for laterally inhomogeneous overburden velocity model. This allows us to analytically investigate the influence of the nonlinear overburden velocity changes on the interval velocity estimations.

Introduction

In this paper, the analytical approach to a moveout investigation in a medium with laterally changing interval velocities is developed. This method allows us to obtain an explicit formula for NMO velocity and to generalise Dix's formula for the medium with laterally inhomogeneous layers and boundaries with small dips. It also allows us to analytically estimate the error of using Dix's formula in the case of not taking into account overburden velocity anomalies.

Blias (1981, 1988) derived the formula for stacking velocities in a multilayered medium with gently curvilinear boundaries and lateral variable velocities in 2-D and 3-D models. This formula allows us to understand the influence of nonlinear lateral changes of the interval velocities and boundaries. Lynn and Claerbout (1982) obtained the formula for stacking velocity for the one layer model. They also considered the inverse problem using obtained second-order differential equation and its numerical solution. Gritsenko and Chernjak (2001) used another approach to solve this equation. Here we should mention that in a layered medium we can see several effects that cannot be seen in the one-layer case.

Here we consider the model composed of N laterally inhomogeneous layers slightly curvilinear boundaries. We assume that normal incident rays are vertical. This assumption is equivalent to considering only the linear influence of the boundary and interval velocity changes on the stacking velocity. Strictly speaking it means the following: Let us consider the model with the boundaries $F_k(x)$ and interval velocities $v_k(x)$. Around CDP point X, we can write the boundaries and velocities presentations in the form:

$$F_k(x) = F_k(X) + G_k(x), \qquad v_k(x) = v_k(X) + u_k(x)$$

where $G_k(x) = F_k(x) - F_k(X)$, $u_k(x) = v_k(x) - v_k(X)$. To consider the linear influence of the boundary and interval velocity changes on the stacking velocity means that we neglect second and larger powers of the values $G_k(x)$ and $u_k(x)$ and take into account only their first power. It means that when we expand any function in a serious of power $G_k(x)$ or $u_k(x)$, we cut all the terms with power more than 1.

For a 1D model (that is the model with horizontal homogeneous layers), we can say that stacking velocities are well understood because we know that NMO velocities are close to RMS velocities and we can calculate interval velocity using Dix's formula.

Dix's type formula for a laterally inhomogeneous layered medium

Let's consider a layered medium with laterally inhomogeneous layers and slightly curvilinear boundaries. Here I will derive the formula for the interval velocity of the last layer assuming that above layers are already known.

As follows from the explicit formulae for NMO velocity (Blias 1981, 1987, 1988), the most influence on the stacking velocities comes from the shallow inhomogeneous layers. It means that in many cases we can neglect the lateral velocity changes in the unknown layer but have to take into account lateral changes in the overlying medium.

Taking this into account, we assume that the deepest layer is locally homogeneous and horizontal but the overlying medium may include lateral velocity changes. These changes (especially nonlinear in the shallow part of the medium) should be taken into account while calculation of the interval velocity for the deep layers. To derive the formula for the deep interval velocity, we combine all layers above the unknown into one medium. We derive the formula for the interval velocity using the second derivative of the CDP time arrivals.

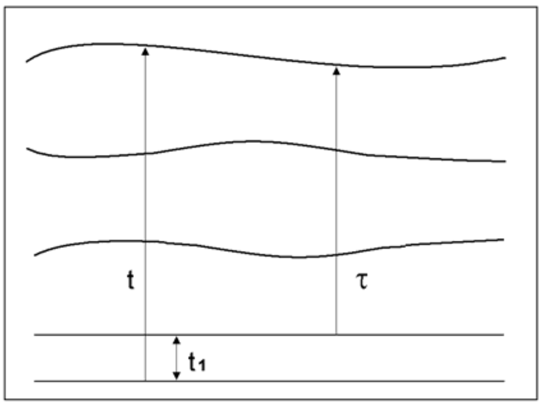

Fig. 1. Time diagram. Interval velocity determination

We consider the last layer to be horizontal, *Fig. 1* and use the following notations: $t_1(\xi)$ is one-way time from the zero-offset reflection point to the top of this layer. $\tau(\xi,x)$ is one-way time along the same ray time from the top of the layer to the measurement surface.

The connection between two way moveout T(x) and NMO (stacking) velocity close-to-zero offset is well known (Goldin, 1986). Combining it with the connection between the second-order derivative of one-way moveout t(x) and two-way moveout T(x) (Chernjak and Gritsenko, 1979), we obtain:

$$V_{NMO}^2 = ½ /(T_0\ \partial^2 t/\partial x^2) \qquad (*)$$

where $T_0 = T(0)$ is two-way zero-offset time. This formula shows that to calculate NMO velocity for close-to-zero offsets, we may calculate the second-order derivative of the one-way moveout function t(x).

To derive the formula for stacking velocity, we use an equation from (Blias et al., 1984)

$$t_{xx} = \tau_{xx} - (\tau_{x\xi})^2/(t_{1\xi\xi} + \tau_{\xi\xi}) \qquad (1)$$

Here t(x) is one-way time from the reflection point to the surface. As we assume that we already know the medium above the last layer, the only unknown value in the equation is $t_{1\xi\xi}$. For the one horizontal layer with the thickness h_n and velocity v_n, we can write the explicit formula for the one-way time $t_1(\xi)$:

$$t_1(\xi) = \sqrt{h_n^2 + \xi^2}\ /\ v_n$$

After differentiating two times with respect to x, we obtain for the vertical ray:

$$t_{1\xi\xi} = 1/(h_n v_n)$$

After replacement $t_{1\xi\xi}$ in the right part of (11) from the last formula and solving the equation we obtain the formula for the product hv. Let us notice that τ_{xx} represents the second order derivative of the one-way NMO function for the bottom of unknown layer.

Let us use the following notation for the *two-way* NMO function: $T_2(x)$ is a two-way moveout function for the bottom and $T_1(x)$ stands for the two-way moveout for the top of the unknown layer. Then $\tau_{xx} = 2T_{1xx}$, $t_{xx} = 2T_{2xx}$. Then (1) leads to the equation for the unknown product $h_n v_n$:

$$2\frac{\partial^2 T_2}{\partial x^2} = 2\frac{\partial^2 T_1}{\partial x^2} - \frac{\tau_{x\xi}^2}{1/(h_n v_n) + \tau_{\xi\xi}} \qquad (2)$$

From this equation we find:

$$h_n v_n = \frac{\frac{1}{2}(T_{2xx} - T_{1xx})}{(T_{1xx} - T_{2xx})(\tau_{\xi\xi}/2) - (\tau_{x\xi}/2)^2} \qquad (3)$$

Taking into account that

$$h_n v_n = \tfrac{1}{2}\Delta T_n, \qquad \Delta T_n = T_{0n} - T_{0n-1}$$

from (3) we obtain a new *Dix's type formula* for the interval velocity of the n-th layer:

$$v_n^2 = (\Delta T_n)^{-1}\frac{T_{2xx} - T_{1xx}}{2(T_{1xx} - T_{2xx})(\tau_{\xi\xi}/2) - (\tau_{x\xi}/2)^2} \qquad (4)$$

Formula (4) includes T_{1xx} and T_{2xx} which can be expressed in terms of zero-offset times T_{0n} and T_{0n-1} and NMO velocities $V_{NMO,n}$ and $V_{NMO,n-1}$ (Goldin, 1986),

$$T_{2xx} = 1/(T_{0n}V_{NMO,n}^2) \qquad T_{1xx} = 1/(T_{0n-1}V_{NMO,n-1}^2) \qquad (5)$$

It means that T_{1xx} and T_{2xx} can be found from real data. The formula (4) also contains the second order derivatives $\tau_{\xi\xi}$ and $\tau_{x\xi}$ for the already known part of velocity model above the last layer (Fig. 1), which can be calculated, using either a method, derived by the author (Blias 1985) or a method by Blias et al., (1984).

For the particular case when all layers are horizontal and homogeneous,

$$\tau_{\xi\xi} = 2T_{1xx} = 1/\sum_{k=1}^{n-1} h_k v_k \qquad \tau_{x\xi} = -2T_{1xx} = -1/\sum_{k=1}^{n-1} h_k v_k \qquad (6)$$

Substituting these presentations into (4), after some transformations, we obtain Dix's formula:

$$v_n^2 = (\Delta T_n)^{-1}(T_{0,n}V_{NMO,n}^2 - T_{0,n-1}V_{NMO,n-1}^2) \qquad (7)$$

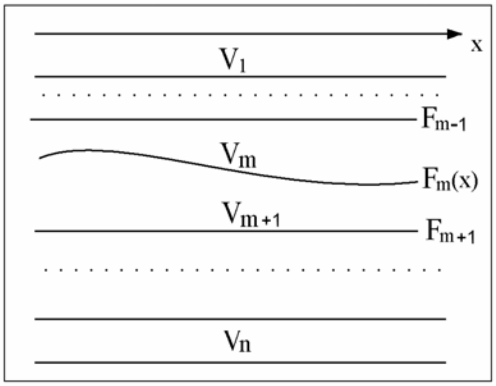

Fig. 2. Model with overburden curvilinear boundary $F_m(x)$

Using *formula (4),* we can derive explicit Dix's type formula for some interesting particular cases. These formulae will allow us to estimate errors of using Dix's formula when we have an overburden velocity anomaly. We will consider two interesting cases with overburden velocity anomalies. In the first case, we assume that the velocity anomaly can be described with a curvilinear boundary. In the second model, we assume that the overburden velocity anomaly is described with lateral interval velocity changes.

Let assume that our velocity model is composed of n layers with all horizontal boundaries except boundary number m, Fig*. 2.*

Using the approach of (Blias 1985), we obtain the formula for the second-order derivative for a one-way NMO function:

$$\partial^2 T_n/\partial x^2 = \frac{1 + F_m''\delta_m (w_n - w_m)}{1 + F_m''\delta_m(w_n - w_m)(w_m/w_n)} \; \frac{1}{w_n} \qquad (8)$$

Here n is the number of the horizontal reflector,

$$w_m = \sum_{k=1}^{m} h_k v_k, \qquad \delta_m = 1/v_m - 1/v_{m+1}$$

From (8), taking into account (*), we obtain formula for near-zero-offset NMO velocity V_{NMO}:

$$\frac{1}{V_{NMO}^2} = \frac{1}{V_{RMS}^2} \; \frac{1 + F_m''\delta_m (w_n - w_m)}{1 + F_m''\delta_m(w_n - w_m)(w_m/w_n)} \qquad (9)$$

We linearize (9) with respect to $F_m''\delta_m$ and obtain (Blias 1981, 1988, 2001):

$$\frac{1}{V_{NMO}^2} = \frac{1}{V_{RMS}^2} \left[1 + (1/v_m - 1/v_{m+1}) F_m''(x)\, a_n\right]$$

where

$$a_n = \Big(\sum_{i=m+1}^{n} h_i v_i\Big)^2 \Big/ \Big(\sum_{i=1}^{n} h_i v_i\Big) .$$

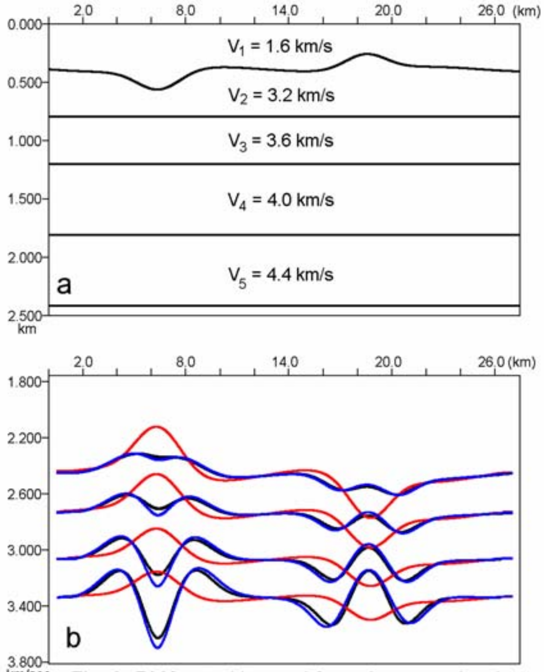

Fig. 3. RMS, stacking and from the second-order derivative velocities for the model with curvilinear boundary

a. Velocity model

b. Stacking (—), RMS (—) and from the second-order derivative (—) velocites

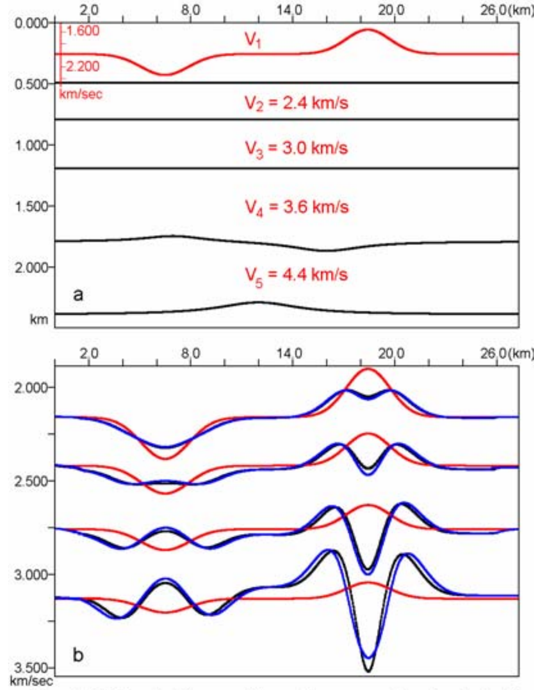

Fig. 4. RMS, stacking and from the second-order derivative velocities for the model with inhomogeneous layer

a. Velocity model

b. Stacking (—), RMS (—) and from the second-order derivative (—) velocites

Let us compare stacking velocities, calculated through the raytracing, RMS velocities and NMO velocities, calculated with taking into account the second-order derivative using *formula (9)*. We consider the model with the strong curvilinear shallow boundary and homogeneous layers, *Fig 3 a*. *Fig. 3 b* shows stacking velocities after raytracing, (----), RMS velocities (---) and NMO velocities, calculated with *formula (9)* (---). This picture shows that RMS velocities are far away from the stacking velocities while NMO velocities for close-zero offset are quite close to the former ones. The only difference between RMS velocities and velocities, calculated with *formula (9)*, is that *formula (9)* takes into account the second-order derivative of the curvilinear boundary. The same result holds for the model with laterally inhomogeneous first layer and flat boundaries, see *Fig. 4*.

From *formula (4)*, with the use of (9), we obtain a **Dix's type formula**

$$v_n^2 = (\Delta T_n)^{-1} (T_{0,n}V_{NMO,n}^2 - T_{0,n-1}V_{NMO,n-1}^2)\, C_n \qquad (10)$$

where

$$C_n = \frac{1 + F_m''\delta_m (w_{n-1} - w_m)}{1 - F_m''\delta_m [1/(2d^2T_n/dx^2) - w_m]} \qquad (11)$$

Formula (10) generalises Dix's formula for a layered medium with a curvilinear overburden boundary $z = F_m(x)$ which separates two layers with interval velocities v_{m-1} and v_m. From (10) and (11), we see that Dix's formula gives a biased estimation. Let us denote by V_D Dix's estimation for the interval velocity, that is the value, calculated with Dix's *formula (7)*. Formulae (10) and (11) show that Dix's velocity V_D can be either more or less the interval velocity. It is well known, that for the horizontally homogeneous layer, Dix's formula for the interval velocity has a bias toward a larger value of average velocity V_{AVE}.

Formula (11) shows that, when there is an overburden velocity anomaly, Dix's estimation of the interval velocity can be less or greater than the interval velocity and this depends on the sign of the expression $F_m''(1/v_m - 1/v_{m-1})$.

We can estimate the error which results from by not taking into account lateral overburden velocity anomalies. Let's assume that

$$2d^2T_n/dx^2 \approx 1/w_n.$$

For the vertically inhomogeneous medium this approximate equality becomes an exact equality. Then, linearizing (11) with respect to $F_m''\delta_m$, we can write:

$$C_n = 1 + F_m''(1/v_m - 1/v_{m-1}) \left(\sum_{k=m}^{n} h_k v_k + \sum_{k=m}^{n-1} h_k v_k\right) \qquad (12)$$

From (10) and (11) we obtain

$$v_n^2 = C_n V_D^2 \qquad (13)$$

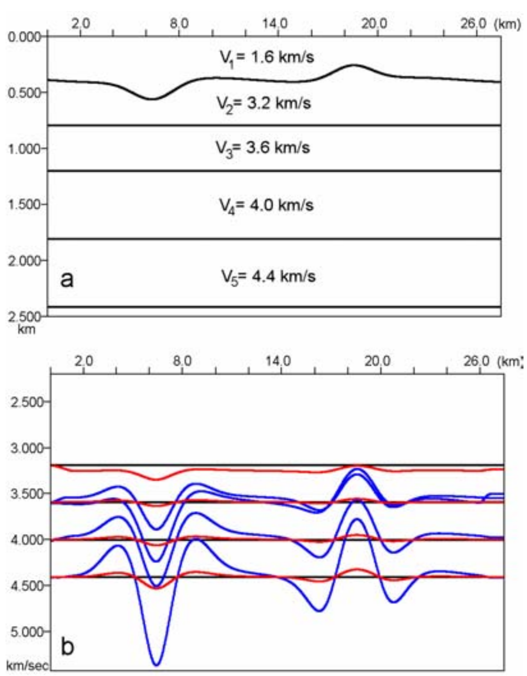

Fig. 5 Interval velocity estimation from Dix's formula and from formulas (9), (10)

a. Velocity model

b Interval velocities: model (—), Dix's (—) and from formulas (9) (10) (—)

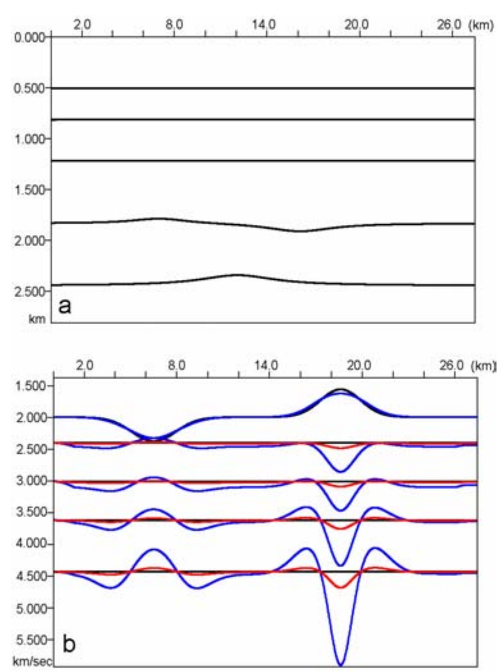

Fig. 6 Interval velocity estimation from Dix's formula and from formulas (13), (14) for model with the first inhomogeneous layer

a Boundaries

b Interval velocities: model (—), Dix's (—) and from formulas (13), (14) (—)

As the scalar C_n describes the difference between interval and Dix's velocity, *formula (12)* shows that the bias of Dix's velocity depends on the sign and the value of the second term. This term, in its turn, depends not only on the transmission properties of the curvilinear boundary (its curvature and velocity difference for the separated layers) but also on the position of this boundary and reflector. The bigger the distance between the velocity anomaly and reflector (the sums in the brackets) the more is the influence of this anomaly.

Formulae (12) and (13) also lead us to an important conclusion: to take into account a strong curvilinear boundary in the shallow part of the section, one has to recover not only its depth but also its second derivative. This explains difficulties in interval velocity determination with overburden velocity anomalies.

Now we will derive a Dix's type formula for a velocity model with horizontal inhomogeneous overburden layer. Let's assume that all layers are laterally homogeneous except layer number m. Using the same method as we used for the medium with a curvilinear boundary, we obtain a *Dix's type inversion formula*

$$v_n^2 = (\Delta T_n)^{-1} \left(T_{0,n} V_{NMO,n}^2 - T_{0,n-1} V_{NMO,n-1}^2\right) B_n \qquad (14)$$

where

$$B_n = \frac{1 + h_m s_m'' \, (w_{n-1} - w_{m-1})}{1 - h_m s_m'' \, [1/(2T_{nxx}) \; - w_m]} \qquad (15)$$

Comparing *formulae (10) and (15)*, we see a complete similarity for these two descriptions of the overburden velocity anomaly: interval velocity v_n differs from its Dix's estimation by the scalar C_n or B_n. Moreover, these scalars C_n and B_n look the same after linearization. It means that an overburden velocity anomaly can be described with two different models.

Let us consider two models. The first model contains the first curvilinear boundary, *fig. 5a*. The second model has the first strongly inhomogeneous layer (*Fig. 6b*) and slightly curvilinear deep boundaries (*Fig. 6 a*). *Fig. 5 b and 6 b* show model interval velocities (---), obtained from Dix's formula (---) and from formulae (10), (11) and (14), (15) respectively (---). We see that taking into account overburden lateral velocity changes gives us much more accurate values of the interval velocities

Two descriptions of overburden velocity anomalies

Let's consider two descriptions of the overburden velocity anomaly. The first model has homogeneous layers divided by the curvilinear boundary $F_m(x)$, *Fig. 2*. The second model includes horizontal layers and laterally changing velocity $u_m(x)$ between the boundaries F_{m-1} and F_{m+1}. Natural condition for these two velocity models is that they keep the same vertical time; that is the zero-offset time is the same. Then it implies that ($s_m(x) = 1/u_m(x)$)

$$[F_{m+1} - F_m(x)]/v_{m+1} + [F_m(x) - F_{m-1}]/v_m = (F_{m+1}-F_{m-1})s_m(x) \quad (16)$$

After differentiating these equations twice, we obtain:

$$F_m''(x)\,(1/v_m - 1/v_{m+1}) = H_m s_m''(x) \quad (17)$$

where $H_m = F_{m+1}-F_{m-1}$. Comparing (11) and (15) for the biasing coefficients C_n and B_n we see that both descriptions are almost equivalent. If we linearize coefficient B_n with respect to $h_m s_m''(x)$, and assume that $2d^2T_n/dx^2 \approx 1/w_n$, we obtain

$$B_n = 1 + h_m s_m'' \left(\sum_{k=m}^{n} h_k v_k + \sum_{k=m}^{n-1} h_k v_k\right) \quad (18)$$

Comparing (12) and (18), we see that the linearized formula for the biasing scalars C_n and B_n are exactly the same if (17) holds.

Here we come to an important *conclusion*: if we use a laterally changing layer to describe an overburden velocity anomaly while this anomaly is caused by a curvilinear boundary, dividing two homogeneous layers, not only zero-offset time but also NMO velocities are almost the same. To illustrate it, we ran modeling for two velocity models, *Fig. 7 and 8*. The thickness of the first layer, two velocities u_1 and u_2 and the interval velocity v(x) satisfy equation (16). This implies that the normal incident times for these two models are almost equal, *Fig. 9 a*. NMO velocities for these two models are very similar, *Fig. 9 b*. This means, that if we don't have a reflection from the curvilinear boundary, we can assume that we have one laterally inhomogeneous layer instead of two homogeneous layers, separated by this boundary. This gives a basis for using laterally inhomogeneous velocities to describe the weathering zone. Then this velocity can be found using an optimization approach (Blias and Khachatrian, 2003.)

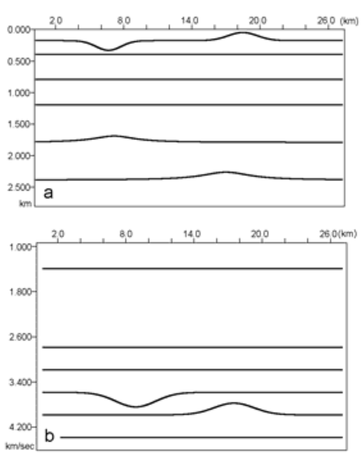

Fig. 7 Velocity model with shallow curvilinear boundary
a - Boundaries
b - Interval velocities

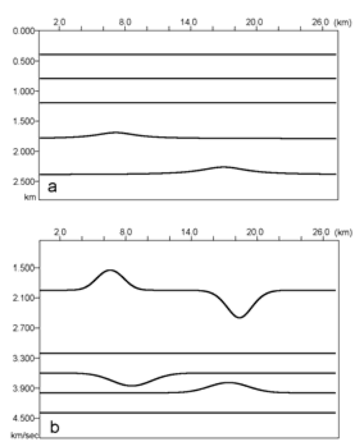

Fig. 8 Velocity model with shallow inhomogeneous layer
a - Boundaries
b - Interval velocities

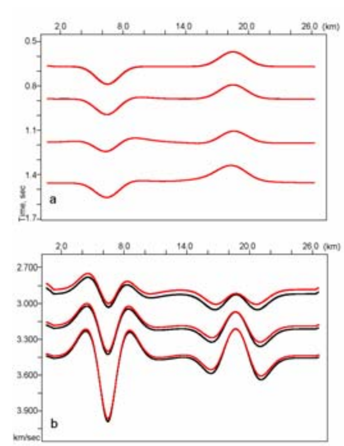

Fig. 9 NMO parameters for two models
a. Zero-offset times for two models
b. Stacking velocities for two models

In this case, the same as when we use a curvilinear boundary, we have to restore not only lateral changes of overburden interval velocity but also its second derivative.

Conclusions

A new Dix's type formula has been derived for the medium with laterally varying interval velocities. This formula includes the second-order derivative of the overburden velocity anomalies. For some particular cases, this formula can be reduced to a simple analytical form which has a clear physical meaning. The correction to Dix's formula is just a scalar, that depends on the anomaly position in the ground.

Two description of shallow velocity anomaly has been studied. Either of this description can be used in traveltime inversion. For a strong shallow velocity anomaly, not only its depth and velocity should be restored, but also the second-order derivatives, that makes deep interval velocity calculation not stable.

Acknowledgments

The author is very much grateful to his wife Ludmila Chavina, who made most of the model calculations and helped a lot to obtain and arrange the results. He also would like to thank Michael Burianyk for his help.

References

Blias E.A. *Approximation of moveout for a layered medium with curved interfaces and variable layer velocities*: Soviet Geology and Geophysics, 1981, N 11.

Blias E, Gritsenko S., Chernjak V.,. *Traveltime derivatives in the layered medium*: Soviet Geology and Geophysics, 1984, N5, pp. 75-81.

Blias E., *Reflected wave's traveltime in the layered media with nonlinear interval velocities*: Problems of Dynamic Theory of Seismic Waves Propagation. Leningrad, N 26, 1987.

Blias E., *Reflected wave's traveltime in 3D layered media with curvilinear boundaries*: Mathematic Problems of Seismic data interpretation. Novosibirsk, 1988, pp. 98-128.

Blias E.A. *Approximated method of determining the trajectories for the rays in 3D layered media*: Soviet Geology and Geophysics, 1985, N 12, pp.82-90.

Blias E., *From stacking to interval velocities in a medium with non-horizontal interfaces and inhomogeneous layers (explicit formulae approach)*: 2002, CSEG convention, expanded abstracts.

Blias E. and Khachatrian V., 2003. *Optimization approach to determine interval velocities in a medium with laterally inhomogeneous curvilinear layers.* CSEG expended abstracts

Chernjak V., Gritsenko S. 1979. *Interpretation of effective parameters of the CDP-method for system or 3D homogeneous layers separated by curvilinear interfaces*: Russian Geology and Geophysics, N 12, 112-120.

Goldin S.V. 1986, *Seismic Traveltime Inversion*. SEG.

Gritsenko S., Chernjak V., *Linearized reflection inversion for the layer with lateral changing velocity*: Russian Geophysics, 2001, Special Edition

Hubral P. and Krey T., 1980. *Interval Velocities from Seismic Reflection Measurements*. SEG.

Lynn W S, Claerbout J., 1982, *Velocity estimation varying media*: Geophysics, Vol. 47, 6.
Popov, M. and Psencik I., 1978, *Computation of ray amplitudes in inhomogeneous media with curved interfaces*: Studia Geoph. Et Geod., 22.