Enhancements to Wave-Equation Multiple Attenuation

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Introduction

Wave-equation extrapolation and adaptive subtraction methods are well established technique for water-bottom and pegleg multiple attenuation (Bernth and Sonneland, 1983; Wiggins, 1988; Julien and Raoult, 1989; Hardy et al., 1991). Conventional implementations of the method, however, can encounter several drawbacks: (1) They behave poorly in the presence of spatial aliasing due to insufficient sampling in the offset dimension; (2) They behave poorly in the presence of pegleg multiples from dipping water bottoms due to multiple emulation not fully honoring real water bottom geometry; (3) They occasionally degrade primaries in the presence of noisy data (which is unfortunately often the case) due to insufficient constraints to the adaptive filter. All of these problems can be mitigated with modifications to the algorithms.

First the application of linear moveout (LMO) in the offset domain prior to the extrapolation reduces event moveout and hence increases the bandwidth of non-spatially aliased signals. Spatial aliasing can be further reduced by first performing trace interpolation using an f-x prediction filter. This is particularly critical when we have a coarse source or receiver spacing. Second, an efficient phase-shift extrapolation algorithm has been used to predict the multiple models, which can handle any kind of 2-D water bottom geometry. Finally, in order to preserve primary events as much as possible, in addition to constraints on the filter, space constraints are also added. None of these enhancements substantially increases the computational effort of wave-equation multiple attenuation.

Multiple Prediction

In a two-dimensional Cartesian coordinate system, wavefield extrapolation by the phase-shift operator can be formulated as

$$P(x,z,\omega) = P(x,z-\Delta z,\omega)e^{ik_z \Delta z} \qquad (1)$$

where $P(x,z,\omega)$ is the frequency-transformed wavefield at horizontal position x, depth z, and frequency ω. Δz is the depth extrapolation step size and k_z is given by

$$k_z = \left(\frac{\omega^2}{c^2(z)} - k_x{}^2 \right)^{1/2} \qquad (2)$$

with k_x the horizontal wavenumber and $c(z)$ the laterally constant water velocity.

With equations (1) and (2), the extrapolation is carried out in a regular grid (x,z), starting from the recorded surface, and ending at some datum below the lowest water bottom point. Energy extrapolated in the area below the water bottom is eliminated from the solution by applying the simple filter

$$P(x,z,\omega) = P(x,z,\omega)\, filt(x,z) \qquad (3)$$

where the filter $filt(x,z)$ is one inside the water medium and zero below the water bottom.

The procedure can be applied both in the shot domain or receiver domain. The only additional computations required by the above scheme over the conventional phase-shift method is the multiplication of the extrapolated wavefield by the zero/one filter [equation (3)] at each depth. As indicated by equation (3), the calculation is not part of the extrapolation. The method can therefore be applied to different types of extrapolation algorithms, including algorithms that handle lateral velocity changes.

Adaptive Subtraction

Filtering of modeled multiples should compensate for inaccuracies or simplifying assumptions in the models, such as approximately estimated water bottom depth, neglected angular dependence of the water bottom reflection coefficient, spherical divergence effects, and the assumption that the water bottom acts as a single reflecting interface (Monk, 1991; Kostov and Nichols, 1995). In the following we will discuss an adaptive subtraction method applied on individual gathers, where in addition to the constraints on the filter applied by Monk (1991), spatial constraints are also exploited.

According to Monk (1991), a model trace $M(t)$ is assumed to differ from the original trace $D(t)$ because the wavelet on the model trace has a different amplitude and phase, and is time shifted. The data trace can be expressed as

$$D(t) = w_1 M(t) + w_2 M'(t) + w_3 m(t) + w_4 m'(t) \qquad (4)$$

where $M'(t)$ is the derivative trace of $M(t)$, $m(t)$ is the Hilbert transformed trace of $M(t)$, $m'(t)$ is the derivative trace of $m(t)$, and $\{w_1, w_2, w_3, w_4\}$ are a set of weights determined by the time shift, phase rotation, and amplitude change. They can be estimated in a least-squares sense to minimize the energy in the result after subtraction of the modified multiples from the input within time and offset windows in each gather.

In practice our model for the matching filter has the following features: 1) A global constant time delay for all the traces in the gather compensates for first order errors in the specification of the water bottom depth; 2) A local 3-component filter optimized for overall time shift, amplitude scalar, and phase angle to convert model traces into the corresponding original traces, compensates for residual errors due to inaccuracies in the depth of the water bottom, and for angle dependent effects of the water bottom reflection coefficients; 3) Quatitatively, the spatially averaged crosscorrelations between primaries in the data and modeled multiples tend to cancel out, while crosscorrelations between multiples sum coherently. Therefore, the component factors $\{w_1, w_2, w_3, w_4\}$ applied to the model traces prior to subtraction from the original are computed from the averaged autocorrelation and crosscorrelations.

Linear Transform

Application of LMO involves the following coordinate transformation

$$\begin{aligned} t' &= t - \alpha x \, , \\ x' &= x \, , \\ z' &= z \, , \end{aligned} \tag{5}$$

where t is two-way reflection time, x is the offset distance between source and receiver, z is depth, and α is a parameter that governs the amount of LMO applied to the data. The variables with primes are the LMO-transformed coordinates. In the skewed coordinate system that results from the LMO adjustment, moveout is reduced, thereby alleviating or removing the spatial aliasing. The angular change before and after LMO correction has a simple relationship to the moveout parameter α. Consider a linear event defined by $t = \beta x$. Measured from the x axis, the event subtends an angle φ such that $\tan(\varphi) = \beta$. After LMO correction, the event has moveout $\beta - \alpha$ and the corresponding angle φ' is defined by $\tan(\varphi') = \beta - \alpha$.

The LMO parameter α determines the amount of adjustment applied to the gather. Because α is constant, invariant in times and offsets. LMO with some choices of α can introduce spatial aliasing of originally low-moveout events while reducing spatial aliasing of originally large moveout events. Thus α must be chosen to avoid such overcorrecting of the input gathers. One rule of the thumb is to choose α so that the amount of applied linear moveout is the average of the minimum and maximum numerical moveout in the input data.

Data Examples

In order to make a multiple prediction we need to know the water bottom depth at each shot and receiver locations along the line. We find that in most cases an accurate estimation of water bottom can be directly obtained from the seismic data. This is one reason we prefer extrapolation over inversion in wave-theory based methods. We first migrate the near traces with the water velocity and then convert the onset time of the migrated water-bottom reflection into the depth of water bottom. The velocity employed for conversion to depth is the same that is used later in wavefield extrapolation.

To avoid truncation effects it is necessary to extrapolate and taper the gather laterally before wavefield extrapolation. In particular it is important to properly model from an energetic point of view major events on the gather (in this case the water bottom and pegleg multiples) beyond the near and the far traces. New traces beyond the near trace are added to the gather by transforming the near trace to other offsets. In the transformation it is assumed that the velocity is equal to water-layer velocity on the entire trace. Traces beyond the far trace are obtained in similar way. Finally, a suitable smooth tapering function is used to taper off the added traces.

The method described here has been applied to a data set from the offshore eastern Canada. During the presentation we will compare

the stacked sections and the selected CMP gathers before and after the wave equation multiple attenuation.

Conclusions

The shortcomings of the conventional wave-equation multiple attenuation method can be alleviated by modifications to the fundamental algorithm. Spatial aliasing in the offset dimension can be reduced by application of linear moveout prior to wavefield extrapolation. Then the wave extrapolation can be efficiently performed with the modified phase-shift algorithm, which can handle irregular water-bottom problems. Further a multi-channel 3-component filter allows introduction of constraints within the adaptation scheme to preserve the primaries as much as possible.

Real data examples demonstrated that water bottom multiples and peglegs are well attenuated over a large range of reflector depth and dips.

References

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